Universität Hamburg

Fakultät für Erziehungswissenschaft Psychologie und Bewegungswissenschaft

ICTMA 14

27-31 July 2009

14th International Conference on the Teaching of Mathematical Modelling and Applications

University of Hamburg

ABSTRACTS

Plenary lectures

Can Modelling be Taught and Learnt? Some Answers from Empirical Research

Werner Blum

University of Kassel, Kassel, Germany

Discussant: Marcelo de Carvalho Borba (Sao Paulo State University, Rio Claro, Brasil)

The lecture consists of four parts:
In the first part, the terms used in this lecture (modelling, modelling task, modelling competency, word problem) are recollected from a cognitive point of view by means of examples. Furthermore, reasons are summarised why modelling is an important and also demanding activity for students and teachers.

In the second part, concrete examples are given of students' difficulties when solving modelling tasks, and some important findings concerning students dealing with modelling tasks are presented.

The third part concentrates on teachers; examples of successful interventions are given, as well as some findings concerning teachers treating modelling examples in the classroom.

In the fourth part, some implications for teaching modelling are summarised: considering quality criteria, encouraging multiple solutions, using a broad spectrum of intervention modes, and fostering students' solution strategies. Eventually, some encouraging (though not yet fully satisfying) results on the development of modelling competency from our own empirical investigations in the context of the project DISUM are presented.

Models and Modeling: Perspectives on Teaching and Learning Mathematics for the 21st Century

Helen M. Doerr

Syracuse University, Syracuse, USA

Richard Lesh

Indiana University, Bloomington, USA

Chair: Mogens Niss (Roskilde University, Roskilde, Denmark)

Model-eliciting activities (MEAs) were developed first and foremost to be used as research tools, not as instructional treatments. In particular, they were designed to: (a) investigate what it means for students or teachers to understand important mathematical concepts, and (b) document and assess how these understandings develop. So, it is largely a pleasant and serendipitous surprise that the byproducts of this research have proven to provide powerful tools for teaching and learning. We will describe how research involving MEAs has shown that, in order for mathematical concepts and abilities to be useful beyond school, new levels and types of understandings are needed beyond those that have been emphasized in traditional textbooks and tests – as well as traditional research on problem solving. We also will describe how MEAs can be used to support the kind of model development that is at the crux of students' understandings of the most powerful ideas in any given mathematics course - and why model development sequences that begin with MEAs often lead to the develop of understandings that exhibit extraordinary levels of transferability, adaptability, and durability.

A Models and Modeling Perspective (MMP) emphasizes the fact that, in virtually every field where learning science researchers have investigated what it means to develop expertise in a given area, it has become clear that experts not only do things differently but they also see (or interpret) things differently. A modeling perspective on teaching emphasizes that teachers' interpretative systems (or models) must include powerful models of students' ways of thinking and their modeling abilities. Furthermore, MMP research is based on the notion that one of the most powerful ways to promote teacher development is to provide experiences in which teachers iteratively express, test and revise their ways of thinking about students' models. Results from such teacher-level modeling studies suggest that teacher-level knowledge and abilities consist of a great deal more than the kind of beliefs, dispositions, and pedagogical content knowledge that have been emphasized in past research on teacher development.

Drivers for mathematical modelling: pragmatism in practice. In touch with the real world!

Christopher Haines

City University, London, UK

Discussant: Katja Maaß (University of Education, Freiburg, Germany)

It is difficult to say what is 'the best way' to introduce certain topics and the more so where mathematical modelling is concerned. Such a way, if it exists, must depend upon a multitude of complex interacting factors, but for the teacher, learner or researcher being *in touch with the real world* is certainly a key factor.

Understanding the processes deployed by students when faced with real-world problems for which practical outcomes might be achieved by constructing a mathematical model has been the subject of a great deal of research. It is common to represent such behaviours in terms of activity within a modelling cycle but not all fit such a model; students exhibit non-linear behaviours and even within such representational cycles they can, and do, follow individual modelling routes.

Against this background, with competing and varied drivers for mathematical modelling locally, and recognising issues of assessment, the following questions are considered.

1. How well do students link mathematical knowledge to the task at hand?
2. How far away is the real world?
3. Is mathematical modelling a driver for mathematical modelling?

Although much of the discussion is embedded in higher education, examples and conclusions readily transfer across all sectors

Mathematical Modelling and a New Role for Mathematics as a Key Technology

Helmut Neunzert

Fraunhofer-Institut für Techno- und Wirtschaftsmathematik, Kaiserslautern, Germany

Discussant: Jens Struckmeier (University of Hamburg, Hamburg, Germany)

Mathematical Modeling and a New Role of Mathematics as a Key Technology Mathematics is one if the core competences in developing reliable and efficient simulations for technical, economical and biological systems; thereby, mathematics found a new role as a key technology. In order to simulate any process, it is necessary to find an appropriate model for it and to create an efficient algorithm to evaluate the model. In practice, still one of the main restrictions is time: If one wants to optimize the process, the simulation must be very fast and, therefore, model and algorithm must be looked as a whole and, together, made as efficient as possible.

Describing two real world problems - air spinning processes and the polishing of jewels - in some details, the lecture will try to demonstrate some modeling aspects, which are crucial in practice:

(a) A problem finding competence, i.e. the capacity to discover real world problems, which may be solved successfully by simulation (this seems not to be well developed in teachers);

(b) To develop a hierarchy of models, which, together with

(c) To construct, for each model, the most efficient evaluation algorithm, allows us to reduce the simulation time;

(d) To check the reliability of the simulation, its limitations and possible extensions; there is never an end in modeling a real world problem.

The lecture ends with a few remarks of a "practitioners" about teaching modeling; I believe, it may be learned, but cannot be taught in a usual way.

Modeling cannot be learned by reading books or listening to lectures, but only by doing. But exactly this doing fascinates students: " *Ich kann wirklich echte Probleme selbststaendig loesen*" (I am really able to solve genuine problems on my own).

Applying Metacognitive Knowledge and Strategies in Applications and Modelling Tasks at Secondary School

Gloria Stillman

University of Melbourne, Melbourne, Australia

Discussant: Rita Borromeo Ferri (University of Hamburg, Hamburg, Germany)

The importance of reflective metacognitive activity during mathematical modelling activity has been recognised by scholars and researchers over the years. Recent research confirms this metacognitive activity even when students as young as 13-15 years are becoming modellers. The application of metacognitive knowledge and strategies in the modelling and application work of Year 9 to 11 students in several Australian schools will be discussed. In particular, metacognitive activity (or lack of it) associated with transitions between stages in the modelling process – particularly in relation to the identification and release of blockages to progress – will be considered.

Not all metacognitive acts are desirable or productive. Productive metacognitive acts can be seen as being at three levels: firstly the recognition of particular strategies as relevant, secondly the choice of a particular strategy for implementation, and thirdly the successful implementation of the strategy. The first decision is preceded by an appraisal by the individual of her/his own knowledge and competence in relation to the task; the second involves an assessment by the individual with respect to the viability of alternatives, while the third is impacted by various sub-competencies of the task solver in relation to the identification and correction of intermediate errors, in addition to ultimate procedural efficiency in obtaining a successful solution.

Metacognitive success was associated by Goos with productive responses to what she termed red flag situations. Red flag situations occur when there is a lack of progress, errors occur and are detected and anomalous results arise. In modelling these situations could result in blockages. Responses by students to such situations could take the form of metacognitive blindness, metacognitive vandalism, metacognive mirages or metacognitive misdirection. *Metacognitive blindness* occurs when a red flag situation is not recognized, so no appropriate action is taken. *Metacognitive vandalism* occurs when the response to a perceived red flag involves taking drastic and often destructive actions that may not only fail to address the issue, but alter the task itself. *Metacognitive mirage* describes a situation when unnecessary actions are taken that derail a solution, because they perceive a difficulty (or difficulties) that does not exist. *Metacognitive misdirection* describes the common situation of a potentially relevant but inappropriate response to a perceived red flag that represents inadequacy, rather than vandalism. Illustrative aspects of metacognitive activity that has been identified in classrooms within which the learning and teaching of mathematical modelling and applications is being enacted which highlight the above will be the focus.

Panel discussion
Modelling perspectives around the world – State-of-the-art

Panelists:
Jonei Cerqueira Barbosa (Brasil)
Morten Blomhoej (Denmark)
Peter Galbraith (Australia)
George Ekol (Canada / Uganda)
Toshikazu Ikeda (Japan)
Pauline Vos (Netherlands)

Chair: Gabriele Kaiser (Germany)

The following aspects will be discussed:

- State-of-the-art on the teaching of modelling in the country or region of each panelist:
- Identified barriers or obstacles for the inclusion of teaching and learning mathematical modelling in mathematics education in the country or region of the panelists;
- Important new challenges to the pedagogy, the teaching and learning of applications and modelling in the country or region of the panelists.

Parallel sessions

FROM MODELLING IN MATHEMATICS EDUCATION TO THE DISCOVERY OF NEW MATHEMATICAL KNOWLEDGE

Sergei Abramovich, Gennady A. Leonov

[a]*State University of New York at Potsdam, USA*
[b]*St. Petersburg State University, Russia*

This paper demonstrates how the integration of modelling perspective into a familiar educational context of discrete mathematics allows for the discovery of new knowledge. (This approach should not be confused with "the interactive development of mathematical knowledge in the social context of everyday teaching" [1, p. 12].) Proceeding from the representation of Fibonacci numbers through a second-order difference equation, the paper explores its two-parametric generalization [2] using computer algebra software and a spreadsheet as modelling tools.

More specifically, the paper investigates the difference equation $x_{k+1} = ax_k + bx_{k-1}$, $x_0 = x_1 = 1$. It introduces a new concept, called Fibonacci-like polynomials, arising from the equation when $a^2 + 4b < 0$. These new for mathematics polynomials possess a number of amazing properties. Most importantly, they can be used as models in investigating a cyclic behaviour of the ratios x_{k+1} / x_k, something that may be interpreted as a natural generalization of a classic exploration leading to the notion of the Golden Ratio. In establishing the existence of such cycles, one can come across a multitude of repeated sets of generalized Golden Ratios where the cardinality of each such set can be as large as one desires. Furthermore, as the results of modelling indicate, the behaviour of the cycles can be universally described in terms of enumerative combinatorics.

From an epistemological perspective, this paper demonstrates the didactic significance of the duality of experiment and theory in exploring mathematical ideas. It shows that whereas one needs theory in order to make sense of a modelling experiment, one can also benefit from the use of technology as a means of validation of concept formation. Illustrations of that kind are of a great pedagogical importance as they aid in motivating the study of mathematics by diverse learners, including prospective secondary teachers.

References.

[1]. Steinbring, H. 2005. *The construction of new mathematical knowledge in classroom interaction*. New York: Springer.

[2]. Abramovich, S. & Leonov, G. A. 2008. Fibonacci numbers revisited: technology motivated inquiry into a two-parametric difference equation. *International Journal of Mathematical Education in Science and Technology*, *39*(6), 749-766

Using CHAT to analyze the collaborative developing and designing process of small mathematical modelling project in upper secondary school

Jonas Bergman Ärlebäck

Linköping University, Sweden

In this paper I use and discuss the theoretical framework of Cultural Historical Activity Theory (CHAT, or sometimes just Activity Theory for short) to analyze the collaborative development and design process of small mathematical modelling projects carried out by two upper secondary mathematics teachers and one didactician. Mathematical modelling is getting a more prominent position in the written Swedish national curriculum documents, but research indicates that the teaching of mathematics at the upper secondary level more or less exclusively is determined by the content of traditional textbooks generally not elaborating on modelling. To work on mathematical modelling with upper secondary students a developmental project involving two teachers and a didactician developed and designed small projects in line with the present curriculum.

Situating this problématique in a cultural and historical context, first the motive of the activity at the community level is taken to be the students' learning of mathematical modelling for the good of society and their own individual, both private and professional, life. The action to realize this activity is the developing and designing of the small modelling projects with the conscious and specific goal to explicitly expose mathematical modelling to upper secondary students. The projects are implemented in mathematics classrooms through (partly) standardized and routinized operations.

Second, Engeströms' model of human activity is used to identify and structure the rules, communities, and division of labour influencing how the two teachers and the didactician (the subject) use different instruments to expose mathematical modelling to the students (the object) resulting in the developed and designed small projects (the outcome).

In CHAT the notions of contradictions and tensions are important since when made conscious or explicit within the activity, they function as driving forces speeding up changes and development both within and between activity systems. Hence, the model of the developing and designing process is analyzed to find contradictions and tensions in the activity system at four different levels. The tensions and contradictions found are discussed in relations to the research literature on obstacles and barriers for the integration of mathematical modelling in the teaching and learning of mathematics, including teachers' attitudes and beliefs.

In addition, some implications of using, and the compatibility of, CHAT within in the emergent methodological perspective of Design Based Research are discussed.

Mathematical modelling, reflexive thought and teachers' education: an articulation

Lourdes Maria Werle de Almeida

Londrina State University, Londrina, Brazil

This paper tries to articulate the triad mathematical modelling, reflexive thought and teacher education. Initially we defended the idea that the situations of mathematical modelling with the futures teachers can be accomplished, essentially, in two perspectives: the personal formation and the professional formation. In this sense we argued that the reflexive thought represents an interaction link between the personal formation and the professional formation the one that referred.

Starting from the analysis of the involvement of students of a course for future Mathematics' teachers with activities of mathematical modelling, we configured the mathematical modelling as a reflexive practice or as a alternative of unchaining the reflexive thought.

The paper signals that are concordant the road of the mathematical modelling, noticed as a pedagogic alternative in the which we make an approach through the Mathematics of a problem no essentially mathematical, and the road of the reflexive thought characterized by John Dewey as something that takes the student to the investigation.

Therefore, configured the potentiality of the mathematical modelling to motivate the interaction and the creativity, noticed that 'the road' of the reflexive thought and 'the road' of the mathematical modelling are concordant, established that situations of mathematical modelling can be included in a course for future Mathematics' teachers, tacked the idea that the development of the reflexive thought by the future teacher has positive influences on his/her formation, our paper defends that is configured a situation ruled in the interaction, in the creativity, in the possibility to raise higher cognitive landings and that can oppose to the paradigm of the technical rationality for the teachers' education.

The mathematical expertise of mechanical engineers – Taking and processing measurements

Burkhard Alpers

HTW Aalen, Germany

German Universities of Applied Science offer as a distinctive feature a very practise-oriented education. Correspondingly, the mathematical education of engineers should enable students to use mathematical methods for solving practical problems. In order to provide such an education, it is necessary to capture the mathematical expertise a mechanical engineer needs in his or her daily practise. Since it is extremely difficult for a mathematician to understand the work of an engineer by simply watching the work over a short period of time, we identified practical tasks in cooperation with a colleague who worked for several years as an engineer in the car industry. We then let students in their final semester work on the task. By reading their documents and additional background material and by interviewing the students as well as the colleague we tried to capture the mathematical thinking processes that occurred during the work. We also tried to see where a more mathematical approach might have made work more efficient. The colleague involved provided information on the extent to which the work of the students reflected the real work of junior engineers.

In earlier reports, we investigated tasks which dealt with the construction of a bearing for an ABS box in a car, with the design of a mechanism for a cutting device and with the dimensioning of machine elements in a simple gearing mechanism. This contribution describes the findings concerning a typical measurement task. In our labs we have a test bench for a steering gear where a steering wheel can be rotated and via the servo mechanism of the gear the wheels are moved. The students should investigate the most vulnerable components, measure the occurring stress in these components using available measurement technology and a data processing program and interpret the results.

The study confirms several observations from earlier studies and offers some new insights. The work within a small algebraic model of the measuring device (so-called DMS strain gauges) was at the heart of the task and an understanding of the model was essential to arrange the device in a proper way. The second essential part is concerned with the representation and interpretation of measurement data. Here, the resulting curves in parameter representation had some peculiarities which had to be connected to what was happening in the test bench. Therefore, the detection of curve properties and the interpretation of curves in application terms are essential qualifications.

Modelling chemical equilibrium in school mathematics with technology

Mette Andresen

National Knowledge Centre for Math Education, University College Capital, Denmark

This presentation gives an example of mathematics enrolled in multi disciplinarity, which was recently tried out in Danish upper secondary school. The example is picked out from the pilot of a development project. It aims to demonstrate how the students' awareness of close connections between mathematics and chemistry in this case may strengthen their modelling competency, because the accuracy of the calculations from the mathematical model under different approximations can be assessed by chemical experiments. The content of the teaching example was the following:

Composition of a system of chemical equilibria in a solution can be described by the actual concentration of molecules and ions contained in it. The mathematical model of the system of chemical equilibria consists of a number of equations with the same number of unknowns (concentrations). It is based on the law of chemical equilibrium or the law of mass action, the law of conservation of matter and charge. When the students solve the system of equations by the use of appropriate software (i.e. GeoGebra), a number of sets of solutions is obtained. Exactly one of these is acceptable from a chemical point of view. One example of a system suitable for inquiry is realised in dissolution of silver chloride in aqueous ammonia. The system may be modelled on different levels of complexity, depending of the number of approximations. For example, if the law of conservation of matter and the law of equilibrium are applied twice, 4 equations can be set up to give a medium – complex description involving the four unknown concentrations $\left[Ag^{+}\right]$, $\left[Ag(NH_3)^{+}\right]$, $\left[Ag(NH_3)_2{}^{+}\right]$ and $\left[NH_3\right]$. The students experience through discussions that the model may be simplified or made more complex. For example, chemical arguments may rather obviously point to a decreased number of unknown concentrations which, consequently, gives fewer equations. On the other hand, regarding Ammonium as a base gives rise to four additional unknowns and equations. Thanks to the software, the students are able to solve the systems of equations in each and every case and then, afterwards, they must choose the appropriate set of solutions. Since the concentration of silver ions can be measured, the students have the possibility to asses whether the calculated solutions seem to be accurate.

Technological discussions in mathematical modelling

Jonei Cerqueira Barbosa, Jonson Ney Dias da Silva

State University of Feira de Santana, Brazil

Mathematical modelling is likely to be implemented by organising the students in groups. In order to produce a mathematical model, different types of discussions may take place in the interactions between students and the teacher or students and themselves. The technological discussions are defined as those that address the translation from a daily or scientific situation to mathematical terms. They include making hypothesis, selecting variables, choosing what mathematical topics to use, relating features of the situation to mathematical properties and so on. This is a crucial part of modelling practices while this type of discussion focuses on the production of a mathematical model. As a result, an understanding on that may support the teachers to follow students' efforts in their attempt to build a model. Then, this paper reports part of a wider qualitative study on students' technological discussions. We got empirical evidence from two groups of students approaching modelling tasks extracted from daily situations (one based on an article from a newspaper) through observation and interview. The data analysis had inspiration in grounded theory. The findings strongly suggest the students structured the real situation through mathematical topics already studied. In some degree, they made the properties of the situations as a sort of "shadow" of mathematical properties of the topics they were applying. They also activated their daily knowledge about the real situation in discussions, but they made it secondary. Then our interpretation of the data points out that students are likely to mobilize school mathematics and their own knowledge on real situations in order to discuss about how to approach the problem. However, the school logic looks predominant in their discussions. As an implication, it is recommended that teachers' intervention should motivate students to move technological discussions toward a more real approach.

Pre-service mathematics teachers' development of mathematical models: Motion with simple pendulum

Sinem Bas, Bulent Cetinkaya, Ayhan Kursat Erbas

Middle East Technical University, Turkey

Raising students using their mathematical knowledge and skills to solve real life phenomena is one of the major goals of mathematics education from primary to tertiary level. Supporting mathematical thinking mathematical modelling creates a viable context for this purpose. The goal of this case study is to investigate pre-service mathematics teachers' development of mathematical models related to the concept of motion with a simple pendulum. Descriptions of the iterative and cyclic modelling process were used as the framework for examining pre-service teachers' modelling process (Lesh & Doerr, 2003; NCTM, 1989). The focus of this study is twofold: (a) what factors facilitate and constrain teachers' modelling process in different modelling stages, and (b) to what extent the modelling framework can explain the teachers' modelling process.

Participants were a group of nine pre-service elementary mathematics teachers in their third years in the program. The study was conducted in two phases. First, participants were provided with an initial preparatory activity where they were informed about the modelling activity on simple pendulum and how to use a graphing calculator in such an activity. One week later, students engaged in the modelling activity lasting four hours. Participants worked in groups of three were audio and video recorded while they were working on the activity. For in-depth exploration of cognitive aspects of the modelling process, an hour long semi-structured interviews were also conducted with each of the three groups after they have finished the activity. Participants' worksheets, experiment reports and reflection papers were also used as data sources.

Preliminary analysis of the data indicated that the pre-service teachers generally passed through the stages of a modelling process proposed in the literature with some differences. Some internal and external factors that supported or constrained teachers' modelling process revealed from the findings. Knowledge of conducting an experiment and context of the modelling problem, motivation, use of technology, and group work facilitated pre-service teachers' modelling processes. On the other hand, prior knowledge about mathematical concepts and lack of metacognitive behaviors hindered participants' transition among the stages of the modelling process. The findings of the study can be useful for teachers and curriculum designers in creating teaching and learning environments with modelling by revealing certain factors that considerably affect the modelling process. The implications of the findings related to the extent to which the modelling framework explains the teachers' modelling process will be discussed.

Documenting the development of modelling competencies of Grade 7 mathematics students

Piera Biccard, Dirk Wessels

University of South Africa, South Africa

The need to research modelling competencies in a comprehensive manner has been recognised (Maaß 2006:113). This paper presents preliminary findings from a study investigating the development of mathematical modelling competencies of Grade 7 students. The focus is on the empirical work conducted during qualitative research on twelve students working on modelling problems in group situations over a period of twelve weeks.

Modelling is a mathematical competence and a means of learning significant mathematics in mathematics education. We explore the nature of competence and modelling competencies. The paper emphasises the development of competencies in the group as a whole as these competencies are often observable when students work in groups verbalising and representing their thinking. We elaborate on decisions made regarding the choice of groups, the tasks used and the design and use of qualitative instruments. We describe the process of measuring the competencies using these instruments.

The data collected supports existing research that modelling competencies do develop when students take part in modelling courses (Kaiser 2007: 116; Mousoulides, Sriraman &Christou 2008: 9) and assists us present the metamorphosis of modelling competencies to create a broader picture of these competencies. Furthermore, the findings may assist in introducing a modelling perspective at any school phase level.

References:

Kaiser, G. (2007). Modelling and Modelling Competencies in School. In Haines, C. Galbraith, P. Blum, W. Khan, S. *Mathematical Modelling (ICTMA12) Education, Engineering and Economics.* Horwood Publishers. Chichester. 110-119.

Maaß, K. (2006). What are modelling competencies? In *Zentralblatt Fur Didaktik Der Mathematik (ZDM): The International Journal On Mathematics Education.* 38(2), 113-142.

Mousoulides, N. Sriraman, B & Christou, C. (2008). The modelling perspective in the teaching and learning of mathematical problem solving at the elementary and secondary school level. MTL short report. Retrieved 2008-08-08. www.umt.edu/math/reports/sriraman/2008_Sriraman_12.pdf

Mathematical Modelling in Distance Course for Teachers

<u>Maria Salett Biembengut,</u> Selma Santos, Thaís Mariane Biembengut Faria

Universidade Regional de Blumenau, BRASIL

In this article we present the principle results of a research project whose empirical data was obtained from a distance course of mathematical modelling to teachers and students of math teacher training. The objectives of the research were to analyze the processes and methods organized, and to verify possibilities of how participants could incorporate what they learned into their practice. The Brazilian movement in mathematical modelling in teaching began in the 1970s, but expanded greatly in the 1990s, especially through official federal proposals allowing the restructuring of teacher courses in mathematics. Despite this, many math teachers don't know how to do mathematical modelling in teaching. This was the motivating factor for us to organize this course. In preparation we needed: didactical material and all structures in the web site to teach (e.g., video conference, chats, virtual meeting). Two groups participated: the course team (10 people – professor, three assistants, seven technological assistants) and 29 participants (three professors, 10 teachers and 16 students). We obtained the data from interviews, observations, and participants' questions and difficulties. The results from the research showed that most participants were not prepared for distance courses. The majority did not read the material; they waited to do their proposals until the virtual meetings (every fifteen days), when the professor explained what had to be done. We analyzed the happenings based on complexity theory. The results of this research were significant for understanding that modelling is a good path for those who want to learn, but learning virtually demands more perseverance, and more time. This occurs when the person indeed has the need to learn.

Analysing the functions of mathematical modelling in science as a basis for developing modelling competency in science education

Morten Blomhøj, Tinne Hoff Kjeldsen

IMFUFA, Roskilde University, Denmark

Mathematical modelling and models are integrated in the practice of many disciplines in the natural sciences as well as in medicine, economy and even some branches of the social sciences. Even though the mathematical concepts and methods involved in the modelling activities in these different disciplines often are very similar or even sometimes the same from a mathematical point of view, the functions and status of modelling and models can be very different depending on the discipline and the context. These differences constitute an important didactical challenge when pursuing the development of students' mathematical modelling competency as an educational goal.

The widespread use of modelling and models in the natural sciences is one of the main arguments for teaching mathematical modelling at the upper secondary and tertiary levels of science education. However, in order for the teaching in mathematical modelling to actually be in agreement with this argument of justification the teaching needs to address at least some of the main differences in the functions and status of mathematical modelling and models in science.

Methodological, we apply in-depth analysis of four selected project reports written by students from the interdisciplinary two-year entrance study programme in science at Roskilde University. The selected projects represent different functions and status of mathematical models and modelling found in modern science. Based on the analyses we illustrate and discuss the different didactical challenges involved in supporting students' reflections and critique of on the one hand mathematical modelling with respect to the phases in modelling process and on the other hand to the functions of a model in a scientific investigation process or in a technological decision making process.

Real-World Modelling in regular lessons: An Experiment

Martin Bracke, Andreas Geiger

University of Kaiserslautern, Germany
Goethe-Gymnasium-Germersheim, Germany

The talk introduces an experiment that is currently conducted at Goethe-Gymnasium-Germersheim with 14-/15-years old students. Over a whole school year there are five realistic modelling projects of different duration, which are linked to the standard math curriculum. The first is a guided modelling which introduces the concept of modelling. For the remaining task the pupils are supposed to work in teams with the greatest possible autonomy.

Central questions of the experiment are

- How to integrate modelling projects into regular lessons in a way, which allows for the introduction of selected topics of a whole school year via realistic applications?
- Do pupils accept the intended frequency of modelling phases as a convenient diversion or do they consider them not worthwhile (concerning the effort)?
- To which extend can we expect pupils to learn mathematical modelling throughout frequent repetition?
- Are contents learned in modelling projects learned more deeply? In other words, what about sustainability of knowledge and skills gained through mathematical modelling in the long term?

During the whole year there is a continuous evaluation through questionnaires (accomplished after each project). The evaluation is designed to take into account gender specific questions as well as the evolution of modelling competencies over time.

Modelling tasks at the teacher-online portal "Program for gifted"

Matthias Brandl

University of Augsburg, Germany

The teacher-online portal "Begabte fördern" (Program for Gifted) was set up by the chair for the didactics of mathematics at the University of Augsburg in the science year of mathematics 2008. The published materials are developed for the support of mathematically gifted students both in the context of extracurricular study groups and within-class grouping. There are several topics available already.

We present three examples from this variety that arise from application and therefore emphasize the process of mathematical modelling. Additionally, the examples are classified by the "Augsburger Modell für mathematische Begabung" (Augsburg model for mathematical ability).

The first example starts with the simple question for the maximum volume of a cone-shaped champagne glass for fixed length of the cup. Starting with a row of real paper cones with different opening angles, the students are led to the non linear relation between the volume of the cone and its opening angle. This aspect again shows up in the process of mathematical modelling by deduction of the corresponding formula. First by curve sketching and then by the application of a computer algebra system the desired cone is identified. The unit was carried out successfully in a 10^{th} grade Gymnasium.

The second example is motivated by an (again initially simple) question concerning the lottery: "Is it more likely that there are more winners of the (6 out of 49) lottery when there are more participants?" The intuition says "Yes, of course.", but the mathematical correct answer is not easy for students at secondary level as the model is a combination of a hypergeometric and a binomial distribution. A closer look on the problem leads to further interesting aspects (like a connection to the Pascal triangle) to be investigated by the students.

At last we present a learning unit that is strongly connected to financial investment business. The student is confronted with a rather open problem. As a decision maker of a consulting group hired by the fund management of an investment company his team members have developed different ways of visualization the given data (i.e. an excel table containing historical stock performances of insider buys within a month ranked by an in-house scoring system to be evaluated). Which way of visualization suits the problem best – and how is it done? Does the scoring system work properly? These questions are to be tackled and answered.

Modelling of various distribution problems with the help of stochastic networks

Thorsten Braun, Engelbert Niehaus

University of Koblenz-Landau, Campus Landau, Institute of Mathematics

Stochastic networks serve as a mathematical model to describe spatial distribution problems. An example for such a distribution problem can be the distribution of resources such as relief supplies or emergency power generator between different places or the movement of people which are infected with a virus to get information about the spread of the virus agent. For our mathematical model it is equal if the model represents the movement of resources or of people.

A stochastic network consists of a set of edges and nodes. The edges are directed and connect the several nodes, whereas not all nodes are connected with each other and not all edges do have back direction. There is a weight attached to every edge, which is a real number out of the interval from 0 to 1. The name stochastic is based on the requirement of a probability space, which prescribes, that the sum of the weights of all edges which start at the same node are adding up to 1.

In our context of application the nodes represent places in a landscape and the edges represent the ways or roads between these places. The weights which are attached to every edge symbolise how much resources or infected people leave that node on this way by percentage.

The modelling process with the students in upper secondary level is problem oriented. Problem-based learning is a student-centred instructional strategy in which students collaboratively solve problems. Learning is driven by challenging, open-ended problems and teachers take on the role as "facilitators" of learning. In our case this means that the learning of mathematical modelling is determined by the distribution problem.

After the teacher has presented the distribution problem the students develop ideas to solve the problem on their own. Maybe the students have ideas for which they do not know the mathematical tools, so it is the job of the teacher to derive the suggested solutions of the students to mathematics. For example multidimensional mapping is not part of the curriculum in school but it is necessary for this problem solving. Altogether for solving the distribution problem the students need mathematical tools which they learned in lower and upper secondary level and mathematical issues from the university, which the teacher has to explain to the students. In some cases it make sense to do the calculations of a mathematical tool with the help of a computer algebra system like maxima or a spreadsheet like openoffice calc, for example when the students have to do matrix multiplications.

Modelling Tasks: Not just Learning Tasks but Deepening Understanding of Mathematics and its Communication

Jill Brown, **Ian Edwards**

Australian Catholic University, Luther College, Australia

This paper reports an intrinsic case study of two students, Tabitha and Tanya, examining their responses to two modelling tasks—one undertaken in Year 9 and another in Year 11. Both students had the same teacher, Peter, in Year 9 and Tanya again in Year 11. Both Tabitha and Tanya achieved particularly well in mathematics throughout their schooling. The first task is an open modelling task designed by Peter. However, as noted by Peter, constraints of external assessment by examination loom large in upper secondary; hence the second task designed by the first author was more structured modelling. Data for this analysis includes student reports for both tasks and for the second task recordings of graphing calculator screens used by students during modelling, post-task interviews, and for Tabitha audio and video recording of her during the modelling task.

In a modelling task, the student's goal is to make sense of the situation so it can be mathematised in ways that are personally meaningful. In doing so, prior knowledge of the situational context can be drawn on but often in idiosyncratic ways. In the first task, Tommy Tinn's Trout Farm, environmental issues and concerns impinge on the students' economic models, whereas in the second task, Save the Platypus, environmental issues and concerns impinge on population models. The work of Stillman (2000) on the impact of students' use of prior knowledge on their approaches to solving real world tasks and Busse's (2005) typology of individual ways of dealing with context are used in the analysis.

English and Doerr have previously "shown how the use of modelling tasks that foster different ways of thinking within a social context provide rich opportunities for teachers to learn about and learn from their students' emerging mathematical ideas (2004, p. 221). A second focus of the analysis is the demonstration of higher order thinking by the students during the modelling tasks as they apply their mathematical and real world knowledge. By higher order thinking we mean instances where there is evidence that a student appropriately (a) makes choices about her solution path (processes, representations, technology use and type); (b) makes links across representations; (c) expects to verify a conjectured solution; (d) appreciates the value of/need for verification; (e) is aware of the value of verification occurring in a yet unused representation or in multiple representations; and/or (f) differentiates between global verification and local checking. Also considered in this analysis are situational factors such as how the context in which the students have learnt mathematics over their years of schooling have impacted on their approaches to these two modelling tasks undertaken at key points in their secondary schooling.

An integrated approach to introducing modelling

Hugh Burkhardt

Shell Centre, University of Nottingham, UK

"Bowland Maths" is a current UK initiative that aims to introduce real problem solving activities into the mathematics curriculum for ages 11 to 16, motivating students for mathematics by showing how it can give them more power over their world. This paper will outline and illustrate the approach, and the many ways in which it is unusual.

The project was funded by the Bowland Trust and the UK Government. Directed by the Trust, it encouraged an exploratory approach to design within a well-specified set of research-based principles. The learning activities are in the form of 3-5 lesson "case studies", in which students investigate a problem situation from the real world or a fantasy environment. The teaching materials were commissioned through a process of open competition. 400 ideas were submitted, 40 detailed proposals commissioned, and 23 case studies from 15 agencies funded. The agencies varied greatly including: a creative teacher, academic design research groups, educational software houses, and media companies. The products, varying from *Alien invaders* through *Reducing road accidents* to *How risky is life?*, cover a wide range of educational and media sophistication. All were independently evaluated and revised.

The package, delivered on a DVD, includes 5 modules of "in school" professional development support, focused on the main pedagogical challenges that the case studies present to teachers of national mathematics curriculum that is largely expository. The professional development modules were developed through a collaboration involving 5 agencies with differentlinds of expertise, led by the Shell Centre. The modules are activity-based. They use a 3-session "sandwich model" in which teachers meet for a structured discussion of the issues and to plan a lesson, then they teach it, then return and reflect on what happened. The single-lesson tasks around which the activities are based present the same sort of modelling challenges as the case studies.

http://www.bowlandmaths.org.uk/ gives something of the flavour of the project and how to access the materials (free only in the UK, alas)

Assessment aligned with these materials, developed by three design groups, will also be discussed and exemplified.

Upper secondary students' handling of real world contexts

Andreas Busse

Ida Ehre Gesamtschule, Hamburg, Germany

It is a widely shared opinion within the scientific and educational community that realistic tasks constitute an essential element in the mathematics classroom. A characteristic feature of this kind of tasks is their embedment in a certain realistic background, the *real world context*.

Whilst it seems to be commonly agreed upon that familiar real world contexts usually have a fostering effect on primary school children the situation seems different for teenage students. Members of this age group tend to interpret given real world contexts more individually so that possible contextual effects cannot be predicted as easily for them as they can for younger children.

In the qualitative-orientated empirical study that is to be presented here, a focus is put on how *upper secondary* students deal with real world contexts. Since these students often handle real world contexts in individual ways, a special methodical approach is necessary. This approach is to allow insight into both, the individual mathematical work on the problem and possible internal processes caused by the real world context. Therefore a three-step-design consisting of observation, stimulated recall and interview was developed, which enables the researcher to reconstruct different levels of action separately although they have taken place simultaneously. This methodical approach is regarded as a triangulation of methods; consequently, the data analysis takes this aspect into consideration.

It was found that a real world context given in a task is not only interpreted very individually but is also dynamic in a sense that the contextual ideas change and develop during the process of working on the task. Furthermore, the data analysis led to four different ideal types of dealing with the real word context: *reality bound*, *integrating*, *mathematics bound*, *ambivalent*. Based on the theoretical background of *situated learning* these ideal types can be understood as effects of – often implicitly given – sociomathematical norms concerning the permissible amount of extramathematical reasoning when working on a mathematical problem.

The results of the study show the importance of an individualised view on students' different ways of dealing with real world contexts. Moreover, the relevance of an explicit teaching of sociomathematical norms in the application and modelling classroom has to be emphasised.

References

BUSSE, A. (2005) Individual ways of dealing with the context of realistic tasks - first steps towards a typology. In: Zentralblatt für Didaktik der Mathematik, 37, 5, 354-360.

BUSSE, A. (2009). Umgang Jugendlicher mit dem Sachkontext realitätsbezogener Mathematikaufgaben. Ergebnisse einer empirischen Studie. Hildesheim: Franzbecker Verlag.

Roles of knowledge in the teaching of modelling at primary school through a French-German comparison

Richard Cabassut, Anke Wagner

Strasbourg University, France
PH Ludwigsburg, Germany

In this article we propose to study in the teaching of modelling in primary school the roles of the mathematical knowledge, of the real world knowledge and of the taught knowledge.

The teaching of modelling is a place where mathematical or real world knowledge is transposed to be adapted to the teaching institution. This double transposition (the mathematical one and the real world one) brings specific difficulties for the teaching of modelling. We will use a French-German comparative approach to contrast these difficulties.

In a first part we will compare the place of modelling in the teaching institution through some observations from both countries in didactic literature, in the curriculum, and in textbooks.

In a second part we will analyse an example of teaching related to the same modelling task, produced in both countries in a primary school class.

In a third part we will compare information given by a piloting course on modelling. This course was realised in France and Germany, following the framework of the European project LEMA (www.lema-project.org). This project offers a teacher training course and resources for the teaching of modelling.

We will point in these parts the problems related to the double transposition of the mathematical knowledge and of the real world knowledge.

Finally we will formulate research questions and conjecture answers to these questions.

References:

Adjiager, R. ; Cabassut, R. (2008) La modélisation dans une perspective de formation et d'enseignement. *Actes du XXXIVème Colloque Copirelem.* IUFM Champagne Ardenne. Juin 2008.

Cabassut, R. (2009) The double transposition in mathematisation at primary school in *Proceedings of 6th Cerme (congress of European society for research in mathematics education).* Lyon. France.To be published.

Maass, K. (2006) What are modelling competencies? *Zentralblatt für Didaktik der Mathematik* 38 (2).

Mathematical modelling of daily life in Adult Education – focusing on the notion of knowledge

Susana Carreira, Nélia Amado, Filipa Lecoq

Universidade do Algarve & CIEFCUL, Escola Básica, Portugal

In this paper we aim to discuss and rethink the notion of knowledge as it is elicited through mathematical modelling of daily life situations, within the context of Adult Education classes.

Some specific questions are guiding our present work: (i) How do adults' social and professional experience and lived reality help them to understand the role of mathematics in daily problems and situations? (ii) Do students from a school program for adults take advantage of their knowledge on daily problems and of their lived experience? (iii) What are the aspects of the modelling process that are mostly affected by the scope of their mathematical literacy and reversely by their social, professional or personal ways of dealing with reality?

The fieldwork consists of a school program for adult education as requirement for certification of middle education. The adult students in this program have generally a very limited mathematical background and reveal a low level of mathematical literacy. Their ages, social and professional contexts are very diverse.

All the work done in classes with the adult students is organised around life issues that are chosen and proposed by the students themselves. Based on their ideas, interests and suggestions, the educators (all school teachers) create and implement different activities, following a set of orientations in terms of units of knowledge and competence development and considering given criteria to fulfil the requirements of certification.

The data gathered from students' observation in the classroom, including documental records, and from interviews (individual and collective) will refer to a two-month period of work on the subject of cookery, which was one of the themes proposed by this class. From recipes to shopping ingredients, from cocktails to the right wine temperature, from salt to healthy food, different problems are considered from several angles, one of them being a "mathematical angle".

Central notions on situated cognition (Lave, 1988; Lave & Wenger, 1991; Rogoff, 1990) will be brought into play to examine the meaning of mathematisation (Freudenthal 1983; Gravemeijer, 2005) and of mathematical modelling competence (Blomhoj & Jensen, 2007). In particular, knowledge – its position and function – in adults' thinking will be at the forefront of our analysis.

Students' modeling cycles in the context of object manipulation and experimentalist mathematics

Susana Carreira, Ana Margarida Baioa

FCT, Universidade do Algarve & CIEFCUL
Escola Básica 2,3 D. Manuel I, Tavira

The present study is a classroom based research where students develop mathematical modelling on real situations that involve experimenting and manipulating with real objects. The research involves two 9th grade classes with students between fourteen and fifteen years old. These students never had this kind of modelling activities in their mathematics classes before.

Our purpose is to discuss the modelling cycles produced by middle school students in an experimentalist mathematics environment – both from the point of view of realistic mathematics education (Gravemeijer 1994) and of the model-eliciting perspective (Lesh and Doerr, 2003).

The modelling cycle as described in mainstream approaches of A&M is a sequence of stages: identification of the real problem, formulation of the mathematical model, production of the mathematical solution or solutions from the mathematical model, interpretation of the solutions, evaluation of the solution in terms of reality and, if necessary, the model is revised and the cycle repeated. Finally, a report with the results and analysis of the problem is produced (Blum & Niss, 1991).

What about experience and the use of concrete objects? Where does such particular type of activity fit in the modelling cycle? In this study we intend to see how hands-on experience in situations that involve using and manipulating objects to solve real problems has a role in students' modelling thinking and in their modelling cycles. Some provisional results indicate that the so-called real model tends to play a considerable part in students' modelling cycles. In particular, the real model and furthermore the manipulation of this real model is seen as a powerful tool to "find" an answer to the problem. In a sense, we are exploring the possibility of seeing experimentalist mathematics as a level of modelling real situations in school mathematics.

References

Blum, W. & Niss, M. (1991). Applied Mathematical Problem Solving, Modelling, Applications and Links to other Subjects – State, Trends and Issues in Mathematics Instruction. *Educational Studies in Mathematics*. 22, 1, pp. 37-68.

Gravemeijer, K. (1994). *Developing Realistic Mathematics Education*. Utrecht: CD-β Press.

Lesh, R. & Doerr, H. (Eds). (2003). *Beyond Constructivism: Models and Modelling Perspectives on Mathematics Problem Solving, Learning, and Teaching*. Mahwah, NJ: Lawrence Erlbaum.

Mathematical modelling skills and creative thinking levels: An experimental study in a China university

Qi Dan[1], Jinxing Xie[2]

[1] *Logistic Engineering College, Chongqing China*
[2] *Tsinghua University, Beijing, China*

In the last decade, extensive experimental studies have been conducted to assess students' modelling skills around the world (Izard et al, 2003). However, as to the authors' knowledge, this kind of research has never been done in China. The current paper tries to fill the gap by introducing the results of a simple experimental study in a China university titled as Logistic Engineering College. Using the test instruments developed in the past years (Izard et al, 2003), we evaluated 33 engineering students in a class, and obtained the distribution of the students' mathematical modelling ability. In order to examine the relationship between the mathematical modelling skills and the creative thinking levels, we also evaluated the students with TTCT (Torrance Tests of Creative Thinking, see for example, Li and Zhang, 1999), and obtained the distribution of the students' creative thinking levels. The data from the experiments show that the correlation coefficient between these two kinds of abilities is 0.815, which reveals that there is a strong correlation between them. We also examined the relationship between their mathematical modelling skills and their scores achieved in university mathematical courses, and found that the correlation between them is insignificant, although some patterns of relationships do exit.

We also conducted an experimental study on whether different teaching patterns of mathematical modelling will affect the skills of mathematical modelling (Ikeda and Stephens, 2003). The data from the research indicate that the average score of the class taught in a discussion way is 1 point higher than that of the class taught in a traditional lecture way, and the difference between these two average scores is significant, which is an evidence that teaching in a discussion way helps to improve the students' mathematical modelling ability.

Izard J, Haines C, Crouch R, Houston K, Neill N (2003) 'Assessing the impact of teachings mathematical modelling: Some implications' in Lamon SJ, Parker WA, Houston SK (Eds) *Mathematical Modelling: A Way of Life ICTMA 11*, Chichester: Horwood Publishing, 165-177.

Li Z, Zhang Z (1999) *Development and Cultivation of Creativity* Beijing: Science and Technology Literature Publishing Press, 305-308. (in Chinese)

Ikeda T, Stephens M (2003) 'Comparing an analytical approach and a constructive approach to modelling' in Lamon SJ, Parker WA, Houston SK (Eds) *Mathematical Modelling: A Way of Life ICTMA 11* Chichester: Horwood Publishing, 201-211.

Modelling the evolution of the Belgian population, eigenvalues and eigenvectors

Johan Deprez

Hogeschool Universiteit Brussel, Belgium
Universiteit Antwerpen, Belgium
Katholieke Universiteit Leuven, Belgium

Matrices are studied in upper secondary and higher education and eigenvalues and eigenvectors in higher education. These topics are interesting from a purely mathematical perspective, but they have numerous interesting and important applications as well. In this paper we describe a series of lessons discussing one of these applications: modelling the evolution of a population using matrices and using its eigenvalues and eigenvectors to study the long term evolution of this population.

We make students aware of the simplifications that are made when constructing the matrix model (for instance: the same birth rates are used each year, immigration and emigration are not taken into account, …). We do this by inserting a phase during which students work with detailed realistic data (obtained from the Belgian statistical institute) to answer a number of questions. Answering these questions, students realize that they have to make certain assumptions that are subject to criticism. Moreover calculations tend to become messy and complicated forcing them to make simplifications. It is only after this experience that the matrix model is introduced.

Then we focus on long term projections of the simplified matrix model. Calculations and graphs show exponential decline of the population. Experience shows that the predicted rapid decline of the population provokes a new discussion on the role of the model and the assumptions made during the construction of the model (e.g. it sheds light on the role of migration). A second observation is the predicted stability of the ‘long term age structure’ of the population. From a mathematical point of view, the model for the long term evolution of a population can serve as an interesting context for the introduction of the concepts of eigenvalue and eigenvector. The ‘long term growth factor’ is the largest eigenvalue of the matrix in the model and one of its corresponding eigenvectors is the ‘long term age structure’. The next step is finding an efficient method to calculate the long term growth factor and long term age structure of a population without having to do massive calculations. This leads to the definition of the concepts of eigenvalue and eigenvector in general, to the well-known methods to calculate eigenvalues and eigenvectors and to a theorem about eigenvalues and eigenvectors of Leslie matrices. Experiences with students in tertiary education (applied economics, mathematics teacher education) will be discussed.

Secondary teachers' beliefs on modelling in geometry and stochastic

<u>Andreas Eichler</u>, Boris Girnat

University of Münster, Germany

Our research report is based on two qualitative studies of teachers' beliefs on teaching stochastics and geometry in secondary school, including the teachers' views on applications and modelling as a part of their individual curricula (Eichler, 2006). Our research is guided by the following assumptions: The teachers' professional knowledge is a significant determiner of both the teachers' instructional practice and the students' beliefs (Chapman, 2001). For this reason, understanding the teachers' knowledge and beliefs becomes a crucial aspect in changing the teachers' instructional practice and in improving the students' knowledge and beliefs concerning mathematical applications and modelling. The theoretical framework of our study is based on the reconstruction of the teachers' instructional planning including both the contents and the goals of mathematics instruction. Data were collected by in-depth-interviews with 22 secondary teachers.

In this report, we address two issues of the contemporary debate on modelling (Kaiser & Sriraman, 2006): 1) The order and the relationship of educational goals focussed on mathematics as a special branch of science, on the one hand, and goals targeting general competencies in model building, on the other. 2) The process of model building itself. Concerning the latter, our research yields four different types of teaching applied mathematics and modelling. We will discuss these four types, of which only one shows high acceptance to modelling that is describable by one of the common circles of model building. The other three types give evidence that, particularly in case of geometry, teaching applied mathematics is restricted to improving students' motivation and to illustrating mathematical concepts, shifting teaching goals from model building competencies to issues internal to mathematics. Further more, we will emphasise differences and similarities of the teachers' beliefs in respect of teaching modelling concerning geometry, stochastics, and mathematics in general. Finally, we will conclude our report by highlighting several obstacles in the teachers' perspectives to integrate modelling in their instructional practice, e.g. questions of assessing model building competencies, legitimising applied mathematics, managing non-mathematical concepts and theories in applied mathematics, or demanding time for modelling.

Chapman, O. (2001). Understanding high school mathematics teacher growth. In Heuvel-Panhuizen, M. (Ed.), *Proceeding of the 25th Conference of IGPME* (Vol. 2, pp. 233-240). Utrecht: PME.

Eichler, A. (2006): Individual Curricula – Beliefs behind Beliefs. In A. Rossman & B. Chance (Eds.): *ICOTS-7 Conference Proceedings*. Salvador: IASE, CD-ROM.

Kaiser, G., & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *ZDM*, 38(3), 302-310.

Understanding and Promoting Mathematical Competencies from Applied Atandpoint

George Ekol

Simon Fraser University, Canada

"What is it that applied mathematicians actually do in application and modelling?" How are these practical experiences "shared", or "taught" at an undergraduate level? Do these experiences bear any semblance to what school teachers need to engage their students in modeling tasks? Mathematicians actively involved in applications and modeling were engaged in learning activities involving technology and dynamic conceptual models. Then they were interviewed on what they do when working on applied modelling tasks. Four modelling competencies are reported from the study: Finding similar examples or phenomena; connecting physical phenomena with visual concepts; building models from the ground up; and communicating broader context. These competencies not only enrich the list of modeling competencies already identified in mathematics education, but they also point to the need for inter or multi-disciplinary cooperation in applied modeling tasks.

From Data to Functions: Connecting Modelling Competencies and Statistical Literacy

Joachim Engel, Sebastian Kuntze

University of Education, Ludwigsburg, Germany

While the dialectic relationship between context and mathematical model is at the core of any mathematical application, there are various approaches to relate reality to mathematics. Some areas of mathematical application are dominated by a structure-oriented approach, where principal considerations and a structural analysis of the context; e.g. by posing some differential equations, lead to a mathematical model. Data as a check of the obtained mathematical result with reality may then enter at the validation step. A different modelling approach includes data from the very beginning, at the beginning of the modelling cycle. This idea follows closely the genetic principle by studying the phenomena of interest first, taking measurements and gradually developing mathematical descriptions of the phenomena. The data-oriented approach, in some way or another, introduces statistical aspects into the modelling process and hence connects statistical literacy with modelling competencies.

We adopt a data-oriented approach to modelling (Engel, 2009) and report on growing statistical competencies of future teachers. The idea of data as a mixture of signal and noise is fundamental in our approach. When learning to model functional relationships students are confronted with the signal-noise idea by looking at the deviation between model and data. Engel, Sedlmeier and Wörn provide empirical evidence that students *implicitly* acquire statistical thinking skills by developing modelling competencies and enhancing their knowledge about functions.

Extensions of these results to other domains - as, e.g. risk assessment or probabilistic concept knowledge – are the focus of an ongoing research project by Engel, Kuntze, Martignon. The underlying competency model for this project has been developed by Kuntze, Lindmeier & Reiss, which relates modelling competencies to statistical literacy. We present theoretical background, research questions, and information about methods and sample. First results of the study are reported and discussed.

References:

Engel, J. (2009): *Anwendungsorientierte Mathematik: Von Daten zur Funktion. Eine Einführung in die mathematische Modellierung für Lehramtsstudierende*. Springer: Heidelberg (to appear September 2009)

SECONDARY TEACHERS' BELIEFS ON TEACHING APPLICATIONS

Frank Förster

Technische Universität Braunschweig, Germany

As a result of the standard based curricula, secondary teachers' beliefs on applications or modelling have developed in the scope of mathematics education in several countries. In contrast, German secondary teachers rarely integrate applications or modelling in their instructional practice. This research report, based on a qualitative study of teachers' beliefs on teaching applications in secondary school (Förster 2008), is focused on the teachers' beliefs that hinder or promote integrating applications or modelling in their teaching practice.

In this report, I will address the following questions:

(a) What are the teachers' reasons to integrate or to ignore modelling in their teaching practice?

(b) Is it possible to identify issues that can explain the gap between the educational demands on modelling and the instructional practice?

Concerning these questions, a qualitative research approach using in-depth interviews with secondary teachers was used. The objective of this approach was to reconstruct the teachers' belief systems concerning applications. The undefined term "beliefs" was specified by the psychological construct of "subjective theories" (Groeben & Scheele 2000). Subjective theories contain the objectives of mathematics teaching and both the teachers' beliefs concerning the nature of mathematics and the teachers' beliefs concerning mathematical applications. A further focus of this research approach was to identify aspects of the genesis of the teachers' subjective theories, in particular in respect to the teachers' own schooling and the teachers' university studies including first practical experiences in teaching at school.

In this report, results with reference to the subjective theories on modelling of teachers will be presented. Further more, some recommendations concerning teacher training will be sketched.

F. Förster (2008): Subjektive Strukturen von Mathematiklehrerinnen und -lehrern zu Anwendungen und Realitätsbezügen im Mathematikunterricht. In: Jungwirth, H./Krummheuer, G. (Hrsg.): Der Blick nach innen: Aspekte der alltäglichen Lebenswelt Mathematikunterricht – Band 2. Münster: Waxmann, 71-103.

Groeben, N. & Scheele, B. (2000). Dialog-Konsens-Methodik im Forschungsprogramm Subjektive Theorien. Forum Qualitative Social Research 1(2). Available at http://www.qualitative-research.net/fqs-texte/2-00/2-00groebenscheele-d.htm.

Modelling in Mathematics Education: some specific characteristics

Regina Helena O. Lino Franchi, Adriana César de Mattos

Methodist University of Piracicaba (Brazil)

We would like to bring up some aspects of modelling in Mathematics Education that does not make sense in Applied Mathematics or teaching of modelling. These aspects are based on our reflection about an experience of modelling. This experience took place in a Mathematics Education Pos-Graduation Course. In Applied Mathematics activity is successful when we get to validate the solutions and to answer the questions accurately. So, the model is very important. It is also important to teach modelling, because it represents a result of the subject we are teaching. But in Mathematics Education, the process of modelling acquires special meaning, since the activity can became an excellent learning environment and develop students potential.

One of the peculiarities of modelling in Mathematics Education is the possibility to modify the original problem during the process, without loss of activity. This freedom, e.g. to change the initial question, makes sense only in modelling in Mathematics Education. In this experience we were supposed to study the correlation among density of population, level of schooling and violence in the city of Piracicaba. We had difficult to obtain data about violence. Then, we decided to modify our question, just studying the relation between population density and level of schooling. The decision was taken by the group, considered that the new question was still interesting in terms of mathematics learning and possibilities of reality interpretation. The first result is to practice modelling and after to reflect and to discuss about the activity from a pedagogical view.

Another specific characteristic observed is that errors made during the process, it can enrich the experience. During the activity process we did two types of errors. First of it was with the data. We used demographic census data and we fixed the income level as criteria to choose sectors to select data. The inconsistency of the data was realised by the students, who knew the city. The data showed an antagonistic description of Piracicaba. However, this fact stimulated the discussion about the social aspects of the problem, which seems quite interesting from student's formation. We looked for the correct data to continue the activity. The second type of error was in the choice of mathematical tools to deal with data. The difference among the educational levels in each sector was revealed by the graphs, according to the participant's opinions about the city. But the deal with absolute numerical data was not efficient. We made several attempts and the good results came from creating an index, called educational level, and using averages to deal with the data.

The first result from this activity is the fact that in Mathematics Education we are free to change the initial question because what matter for us is the process. The second result is about the role of the errors during the process of mathematics learning.

AN INVESTIGATION OF SWEDISH UPPER SECONDARY STUDENTS' MATHEMATICAL MODELLING COMPETENCIES

Peter Frejd & Jonas Bergman Ärlebäck

Linköping University, Sweden

This paper reports on work in progress on a pilot-study aiming to gain insight into the competencies of Swedish upper secondary students working with modelling problems. Generally, research focusing on mathematical models and modelling in the teaching and learning of mathematics in Sweden is sparse, and hence there is a need for research at all instances of the educational system. Resent research on the Swedish national curriculum documents shows that mathematical models and modelling have been gradually gaining emphasis during the last 40 years. However, the notion of mathematical models and modelling is not by any means explicit, but rather described in vague terms. Research also indicates that teachers' may have no or weakly elaborated conceptions of mathematical models and modelling.

This paper investigates mathematical modelling competencies of upper secondary students in the current educational system by using a set of multiple-choice questions, originating from Haines, Crouch and Davis (2001), to get an indication of the present situation. These questions, used and extended by many scholars, aim to measure the students' competencies in different phases of the modelling cycle, were translated into Swedish. A selection of 14 of the 22 original questions was made based on a pre-pilot study. Subsequently, four different tests, each consisting of 7 questions, with some overlapping items, were constructed. In addition, questions about gender, level of mathematics courses taken, grade, an open question about the students' previous experiences with mathematical models and modelling as well as their present conceptions, and, finally, some attitude questions were included in the tests. The tests were sent out to 30 classes all over the country and we estimate that we will have data from approximately 500 students.

The analysis of the data provided by the tests presented in this paper will, besides reporting on the students' competencies on the test items, try to provide students' different conceptions of models and modelling, account for their attitudes towards models and modelling, and give indications about possible connections between students' modelling competencies and their grades in mathematics. The issue of gender and regional differences will also be discussed, as well as the Swedish students' results in relation to results from other studies in different countries using the same questions.

Haines, C.R., Crouch, R.M. and Davis, J. (2000) Mathematical modelling skills: A research instrument, Technical Report No.55, Department of Mathematics. Hatfield: University of Hertfordshire.

Evolution of Applications in Modelling in the Senior Secondary Curriculum in Queensland

Peter Galbraith[1], Gloria Stillman[2]

[1]*University of Queensland, Brisbane Australia*
[2]*University of Melbourne, Melbourne Australia*

In Queensland Australia, mathematical modelling and applications has featured in senior secondary mathematics curricula for two decades. In the latest syllabuses being implemented for the first time in 2009, it is expected that students will model mathematical situations and constructs, solve problems and investigate situations mathematically across a range of topics. The general objectives for modelling and problem solving include the following:

- apply problem solving strategies and procedures to identify problems to be solved, and interpret, clarify, and analyse problems.
- identify assumptions (and associated effects), parameters and/or variables during problem solving
- represent situations by using data to synthesise mathematical models and generate data from mathematical models.
- analyse and interpret results in the context of problems to investigate the validity of mathematical arguments and models.
- develop and use coherent, concise, and logical supporting arguments to justify procedures, decisions, and results.

In this presentation we report on a longitudinal study of the implementation of this initiative, as seen through the eyes of selected teachers and administrators who have been centrally involved in its development and on-going practice. The data consist of responses to structured and open interview questions, syllabus documents, and application and modelling tasks designed and implemented by teachers.

Themes pursued include the following:

1. Perceptions of the main drivers, sustainers, and impediments.
2. Nature of teacher support, both by external means and self-generated.
3. Balance and/or tensions between *applications* and *modelling*.
4. Incorporation of syllabus objectives into classroom activity.
5. Implementation of assessment criteria.
6. Impact of increased technology on the types of tasks developed.
7. Examples of tasks created and implemented.

The content of the presentation will draw from a selection of these themes.

Modifying teachers' practices: the case of an European training course on modelling and applications

Fco. Javier García, Luisa Ruiz-Higueras

University of Jaén, Spain

Within the European Comenius project "Learning and Education in and through modelling and applications" (LEMA), a teacher training course has been designed, implemented and optimized. The course covers a range of pedagogical issues aiming at supporting teachers in the use of modelling and applied tasks in their teaching practices, both from the perspective of *the teaching and learning of modelling* and *modelling for the teaching and learning of mathematics*. Through the course, a change on teachers' beliefs about the nature of mathematics and its teaching is expected.

The connection between teachers' beliefs and teachers' practices constitutes a problematic issue for research in mathematics education. Two different (ideal) perspectives can be observed: on the one hand, that centred on beliefs, assuming that changes in beliefs lead to changes in teachers' practices. On the other hand, that focusing on practices, assuming that changes in teachers' practices come first and they make teachers' beliefs evolve. This distinction can be useful from a research perspective in order to formulate where the focus is. But reality shows us that changes in practices may occur without any significant changes in beliefs and, on the contrary, changes in beliefs may lead to no real changes in teachers' practices.

Different actions have been developed within the project in order to measure the real impact the course is having on teachers' beliefs (Maaβ, 2009). Therefore, in this paper we will focus on the other side, that is, the side of teachers' pedagogical practices.

Within the Anthropological Theory of the Didactics, any human action is modelled as a praxeology. Particularly, a *mathematical praxeology* can be identified when students are facing a problem where some mathematical techniques are used in order to solve it and some discourses explaining and justifying their (mathematical) action can be established. In the same way, when teachers activate pedagogical techniques in order to deal with pedagogical problems emerging in their teaching and a (pedagogical) discourse explaining and justifying their practices can be identified, their teaching activity can be described as a *didactic praxeology*.

In the paper, the Anthropological Theory of the Didactics will be used to identify and analyse what are the didactic praxeologies that the LEMA training course is aiming at developing in participants. Particularly, the *model of teachers' actions* developed by Sensevy et al. (2000, 2005) will be used in order to identify *topogenetic, mesogenetic and chronogenetic* techniques promoted by the LEMA training materials.

Math to Work, Math is Everywhere, Math is More

Sol Garfunkel

COMAP, Lexington, USA

In this talk we will discuss three new COMAP initiatives. While each of these is based in the U.S. they are all generic enough to have natural implications for international settings. The first, Math to Work involves the development of new curricula aimed specifically at the articulation between school (both secondary and tertiary) and the workplace. This effort follows on a three-year project, entitled WorkMap in which a team of researchers and mathematics educators visited 15 workplaces to observe mathematics in use. The point of the WorkMap project was to see what mathematical skills and conceptual understandings were being called upon (specifically by entry-level IT workers) and how those skills and understandings corresponded to the academic coursework typically required by employers. Not surprisingly there was a rather strong mismatch. The goal of the Math to Work project is to help remedy this situation.

The second, Math to Work, represents a new public television and web-based attempt to explain the importance and ubiquitous nature of mathematics and its applications to the general public. The project will create a one-hour prime time television show along with a web-based instructional series geared for an adult general audience. The web material will include short video pieces showing mathematical applications and models in the home, hospitals, schools, offices, farms, city streets, museums, etc. There will be extensive use of applets and game technologies.

The third is a foray into the world of mathematical politics, in the hopes of bringing comity and coherence to the varied approaches to improving mathematics education. While the infamous ‘math wars’ have calmed somewhat, the debate about how to best improve mathematics education continues. While any serious look at the quantitative needs of modern society tell us that improvement is necessary worldwide, politicians still act as if mathematical knowledge is a zero sum game. Our country has to do better on PISA or TIMMS than their country. Our state has to do better than their state if we are to attract more high paying jobs. This is nonsense and we know it. There are simply not enough quantitatively literate people in the world to run the world. It is well past time to talk truth to power. It is well past time to tell truth to ourselves. Math is More is an attempt to mobilize mathematics educators around a platform of sensible and achievable goals.

Real-world modelling in regular lessons: Results and conclusions

Andreas Geiger, **Martin Bracke**

Goethe-Gymnasium-Germersheim, Germany
University of Kaiserslautern, Germany

This talk is the sequel of “Real-world Modelling in Regular Lessons: An Experiment”, which is about a long-term trial on mathematical modelling with realistic problems, conducted at Goethe-Gymnasium-Germersheim with 14-/15-years old students.

First, we motivate the selection of five topics from the standard math curriculum, which are chosen to be accompanied by modelling projects. Then one of the projects is introduced in detail, as well as solutions developed by the pupils.

Next, we discuss some results of the evaluation, which is done through questionnaires (accomplished after each project). Here the main focus will be on the consequences regarding two of the central questions of the overall trial, namely

Do pupils accept the intended frequency of modelling phases as a convenient diversion or do they consider them not worthwhile (concerning the effort)?

To which extend can we expect pupils to learn mathematical modelling throughout frequent repetition?

This discussion also takes into account gender specific questions and includes the results of a comparison project: At the end of the experiment there will be a final modelling project in which pupils of the trial group as well as pupils of another course (unexperienced in mathematical modelling) work on the same project.

Factors affecting teachers' adoption of innovative practices with technology and mathematical modeling

Vince Geiger

Australian Catholic University, Australia

This paper investigates the affordances and constraints that influence the adoption, by teachers, of computer algebra systems (CAS) in order to enhance students' experiences with mathematical modelling. Mathematical modelling – formulating a mathematical representation of a real world situation, using mathematics to derive results, and interpreting the results in terms of the given situation – is a significant element of the senior mathematics syllabuses in Queensland and the Australian Capital Territory, Australia and appears, as applications of mathematics, in the curriculum documents of most other Australian states. CAS enabled technologies not only have the capability to perform a wide range of mathematical procedures, such as, function graphing, matrix manipulation and symbolic operations, but also the capacity for these different facilities to interact. As CAS enabled technologies are developing increasing acceptance in mainstream mathematics instruction there is need to explore and understand the synergies that might be developed between these technologies, current curriculum objectives and to identify implications of these synergies for classroom practice.

While there is significant research related to solving contextualized problems through the use of the multiple representational facilities offered by digital technologies, and substantive argument to support the use of CAS to enhance the process of mathematical modelling, literature that deals with technology mediated student-student interaction is only just emerging. The project reported upon here aims to develop a greater understanding of the factors which affect the uptake and effective use of CAS technology to enhance the study of mathematical modelling within secondary school classrooms and so add to developing theory in this area.

Data is drawn from a one year pilot study of five different senior secondary school classrooms located within two different educational juristicions where the use of technology and the teaching of mathematical modelling is encouraged or mandated in relevant curiculum design documents. Analysis of these data indicates there was wide variation in the degree to which CAS technologies were employed in teaching mathematical modelling. The analysis suggests that teacher beliefs about the nature of mathematics, the role of technology in learning and the relationship between engagement with mathematical modelling and learning mathematics have a vital roles to play in creating a classroom culture that is supportive of technology enhanced mathematical modelling. Further, a teacher's expertise with CAS, in addition to a disposition towards exploratory approaches to learning mathematics, were noted as an influential factors.

Finally, implication of the findings of this paper for theory related to the processes of mathematical modelling will also be discussed.

Geometry: paradox of an applied science a priori

Boris Girnat

University of Münster, Germany

According to Houdement & Kuzniak (2001), geometry can be seen either as an *empirical* science or as an axiomatic theory *a priori*. In the first case, geometry consists of empirical hypotheses on physical objects which can be verified by measurement and experiment. In the latter case, geometry follows the Euclidean ideal of an axiomatic theory, assuming that its theorems can only be proven in a purely deductive manner, entirely independent of empirical results of measurement and experiment. If geometry is regarded in this way, it seems to be questionable how it can be used in model building processes or even applied to real-world situations at all.

Based on the structuralist theory of science, Balzer (1979) has established a theoretical framework to explain the applicability of a geometry a priori, explicitly describing how geometry is factually used in empirical sciences. His results provide a remarkable impact on the main topics of the current debate on modelling, which seem to be *educational goals* and *competing analyses of the model building process* (Kaiser & Sriraman 2006). If Balzer's results hold, didactics will be confronted with some theses remarkably different from commonly shared opinions, as it will be presented in this article: 1) The typical use of geometry in science is an axiomatic one. A restriction to an empirical view, which is factually detectible (Houdement 2007) and sometimes didactically advised, leads to an inappropriate understanding of applied geometry. 2) In case of geometry, none of the existing theories of model building processes matches. The geometrical model building process is fundamentally different from other cases of mathematical modelling, not leading to testable hypotheses and not providing a stage of validation, but mainly focussed on revealing initial conditions. 3) As far as geometry is not concerned in the context of science, but in the scope of everyday life applications, the main contribution of a geometry a priori does not consist in generating new *geometrical* hypotheses, but in testing and refining prescientific *non-geometrical* theories and assumptions.

Balzer, W. (1978): *Empirische Geometrie und Raum-Zeit-Theorie in mengentheoretischer Darstellung*. Kronberg: Scriptor-Verlag.

Houdement, C. (2007). Geometrical Working Spaces, a tool for comparison. *Proceedings of CERME 5*. Larnaka, Cyprus (pp. 972–982).

Houdement, C. & Kuzniak, A. (2001): Elementary Geometry Split into Different Geometrical Paradigms. *Proceedings of CERME 3*. Bellaria, Italy (Web).

Kaiser, G., & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. In *ZDM*, 38(3), 302-310.

Problem Fields of Everyday Life in Mathematics Education

<u>Günter Graumann</u>

University of Bielefeld, Germany

Mathematics education should be a mixture of different methodical proceedings whereat problem orientation should constitute almost one third. But because this is not true in reality of school we have to strengthen problem orientation in mathematics education. According to the frame of ICTMA I will focus on problems concerning everyday life. For this I first will present my conception of "Practice Orientated Mathematics Education" whereat the focus lies on working within realistic situations. Then I will comment on this by explaining five units (expenses of a dinner at home, expenses of buying an automobile or a motor cycle, dyke raising, sound nutrition, architecture in our city and aesthetical aspects in works of art based on geometry) which have already performed in mathematics teaching.

Examination tasks - with modelling problems and use of technology?

Gilbert Greefrath

University of Cologne, Germany

For creating test tasks many aspects should be considered. In particular examination tasks containing modelling problems require special attention.

In the paper the current situation of final examinations in Germany is outlined. There are clear differences between the individual Federal states and a wide range in the single states. Outgoing from the German situation we illustrate two important aspects on creating examination tasks.

The first aspect concerns problems resulting from the real situation described in the task. As a basic principle applications in education should only used in authentic real situations or if they bring advantages in understanding the problem. Due to the complexity factor tasks for the whole modelling process are in most cases not possible. Likewise some mathematical areas (e.g. Analysis) seem more suitable for modelling tasks as e.g. Linear algebra. In each case the necessary competencies for examination tasks are moved from calculating to interpreting and validating. The question remains open, how much modelling is at all possible in a test situation.

The second aspect concerns the use of digital media. The use of digital tools in mathematics education is desirable and necessary. Thereby the use of real data in modelling tasks can be facilitated. For the examination of mathematical competencies, which were acquired in education with digital tools, it does not require necessarily the digital tools in the test situation. This is on more challenge on creating new examination tasks.

REFERENCE

Greefrath, G., Leuders, T., Pallack, A.: Good examination tasks – with or without technology. Gute Abituraufgaben – (ob) mit oder ohne Neue Medien (German), MNU, 61 Bd. 2 (2008), 79-83

Examining Mathematising Activities in Modelling Tasks with a hidden Mathematical Character

<u>Roxana Grigoraş</u> Fco. Javier García, Stefan Halverscheid***

**University of Bremen, Germany **University of Jaén, Spain*

When being involved in modelling processes, students pass through a mathematisation phase, which is mainly seen as the step between mathematics and the rest of the world, when considering Pollak's modelling cycle. The mathematical character of typical modelling tasks might sometimes be questionable, but usually the fact that mathematics can help solving such tasks appears clear from the very beginning. This is typically combined with the fact that these modelling tasks normally are brought into the classroom by a mathematics teacher in a mathematics lesson. Both facts configure a didactic contract where students *know* that mathematics has to be somehow activated. Among other consequences of this situation, a superficial use of mathematics or, even worse, a lack of sense in students' mathematical activity, can be observed.

In this paper, we will explore a partial rupture of this didactic contract: rupture on the side of the task. That is, we will explore how a change in modelling tasks towards a formulation with a hidden mathematical character or, at least, with no clear indications that are aiming to a mathematical solution, could affect students' mathematising activity. While exploring this, other circumstances that configure the didactic contract previously described will be controlled. For instance, it is irrelevant to change the formulation of the task (in the direction mentioned), if the teacher that is posing the tasks quickly introduces some mathematics in the study process (like some data, ideas, hints, questions, etc.). When focusing on how mathematising emerges, we want to understand why students perform in quite different levels, from almost no mathematics involved in their solutions (*weak answers* to the problem) to more or less sophisticated answers.

Two different frameworks will be used and articulated. On the one hand, the Anthropological Theory of the Didactics (Chevallard, 1992, 1999) will be used to describe students' activity when solving tasks without numbers. Particularly, students' mathematising activity will be described in order to have a deep understanding of how mathematical and non-mathematical issues are activated together. The notion of *mixed praxeologies* (Artaud, 2007) will be used to catch the essence of an activity like mathematisation that often involves complex and dense relations between the *real world* and the *mathematical world*. On the other hand, the epistemic model of Abstraction in Context (Halverscheid, 2008) will be considered to structure the students' work on the task.

Revitalizing Sustainable Heuristics through Geometric Modelling

Lenni Haapasalo

University of Joensuu, Finnland

Even though literature analysis reveals a wide cavalcade of researchers' views of the term 'mathematical modelling', the discussion on learning psychological aspects has mostly been neglected. From a constructivist viewpoint, making of mathematics and applying mathematics are basically triggered by the same kinds of mental processes. A long-term study of the history of mathematics reveals eight main motives and activities, which proved to lead very often to new mathematical results at different times and in different cultures for more than 5000 years. This network of the so-called Zimmermann activities can be taken as a framework for the structuring of learning environments and for analyzing student's cognitive and affective variables.

Modern technology can not only promote those eight dimensions of mathematics making but it can also revitalize beautiful mathematical ideas, which have been developed by great mathematicians through centuries. Not long ago, envelopes of curves, involutes, caustics, and parallels, for example, were standard topics for freshmen. The rich concept curve served as an assembler of the isolated parts of mathematics: Geometry, Algebra, Trigonometry, Analytical Geometry, Calculus. As a result of moves towards generalization and rigor, especially in mathematics education, the special curves and most of the vital geometric spark, which has ignited so many minds in the past, have been cancelled. At school, even the study of hyperbolas and parabolas has degenerated into treating them as graphs of functions. Analysis courses at school or university maintain such an unsatisfactory view with even non-obligatory courses such as Differential Geometry using special curves for illustrative purposes only. As a result mathematics teachers are not aware of the educational potency of those curves as a fruitful field for exploration with geometric, kinematic, algebraic and other pre-calculus tools.

The presentation emphasizes with concrete examples how modern technology gives opportunities to plan and realize psychologically meaningful situations, which allow students to be designers of their own learning. A virtual model on computer screen, for example, can very often span for the learner, even for a small child, a more appropriate investigation space (*Entdeckungsraum*) than a conventional "real-world situation". Thus, technology should not be considered as an interface on a one-way street from real world to the world of mathematics. It also enhances and empowers the interpretations in different kinds of representations within instrumentation and instrumentalization processes. As the latter means that the tools shapes the mathematical actions of the user, even more important than to solve a given problem is to promote students to find and pose new own problems related to a situation.

Mathematical Modeling and Practice Teaching with the Relevant Mathematical Courses

Zhonggeng Han[1] **Xiaoli Guo**[2] **Guangji Shao**[1] **Raoshan Qian**[1]

[1]*Zhengzhou Information Science and Technology Institute, Zhengzhou, China*
[2] *Zhengzhou University of Light Industry, Zhengzhou, China*

Through ten years of teaching and practice, we have formed a set of more effective teaching and practice modes for mathematical modeling. It expedites the reform of the relevant mathematical curriculum and cultivates and promotes the innovative ability to applying mathematics. Our work has been approved by many experts, and also won several teaching awards results. The following is our main methods:

Discussing by students as the class style to train the students' individual innovate ability. In the class, students play the major role, talk each other or discuss with teacher equally. Both teachers and students can be put into the interactive teaching system. This can not only greatly muster the enthusiasm of students to study but also changes the mode of "Teacher says, students listen".

Leading activity-oriented groups, cultivating students' innovation ability. Mathematical modeling activity on group-oriented will help students to cultivate the spirits of cooperation. They can exert collective intelligence. We divide the students into small groups to discussion and research the actual question.

Advocating divergent thinking, cultivating imaginative innovation power of students. In the practice of teaching mathematical modeling, there are two main ways to innovate imaginative power of students through solving practical problems using mathematics. One is with a series of questions to expand, the other is by analogy and instinctive ideas to spread up. Secondly by analogy and instinctive ideas to study the question.

Reforming the forms of course work and examination, cultivating scientific research ability of students. We change the traditional school examination which only test what the students learn on classes. Now we divide students into groups to discuss the problems together, and its contents are not limited by class. The extracurricular work to students is the actual problems which are relevant with curriculums.

Combining teaching and computer, cultivating the practical innovating ability of students. In the teaching mathematical modeling process, by using the powerful mathematical software will help to cultivate students' innovative ability.

"Mathematical Modeling is a road to cultivate the students innovative talents and a road to success for a vast number of students."

Modelling and the Educational Challenge in Industrial Mathematics

Matti Heilio

Lappeenranta University of Technology, Finland

Mathematics, modelling and simulation, so called mathematical technology is emerging as a vital resource to achieve competitive edge in knowledge based industries and development of society. This vision about the role of mathematics has inspired efforts to enhance knowledge transfer between universities and industry. Especially it means a challenge for university education. A modern view of mathematics should be reflected in curricula and educational practices.

We should bring the flavour of a fascinating art to the classroom, to convey the vision about mathematics at work, the diversity of application areas and practical benefits. The challenge is to find ways to make the theoretical content transparent and communicate to the students the end-user perspective of mathematical knowledge.

The educational challenge should be visible in curriculum development, up-to-date contents, innovative teaching methods and educational programs. Many innovative practices have been adopted to facilitate the knowledge transfer. Development of modelling education is a crucial part of this endeavour. I discuss the educational challenge from the curriculum level and also as a challenge of university pedagogy of applied mathematics. The actual objective and fascination is the students' exposure to open problems, addressing questions arising from real context. A good modelling course should

(a) contain an interesting collection of case examples which stir students' curiosity
(b) give an indication of the diversity of model types and purposes
(b) show the development from simple models to more sophisticated ones.
(c) stress the interdisciplinary nature, teamwork aspect, communication skills
(d) tell about the open nature of the problems and non-existence of "right" solutions
(e) bring home the understanding of practical benefits, the usage of the model
(f) tie together mathematical ideas from different earlier courses

I refer to case examples of industrial math projects illuminating the educational challenge. I report about developments in the MS curricula and novel practices that have been introduced in Europe. I also report about some recent and ongoing surveys and campaigns related to mathematics-industry interaction.

Why cats happen to fall from the sky sometimes
or
On good and bad models

Hans-Wolfgang Henn

TU Dortmund, Germany

Nature obeys, at least partly, certain mathematical rules. For a long time, people have tried to generate quantitative mathematical models in order to explain these phenomena and thus to permit forecasts for the future. The model mentioned in the title tries to show that these models do not always provide the desired results.

It is an important task for schools to sentisise pupils for the problems arising in mathematical model building situations. After all, it is one goal of mathematics education to educate pupils to become responsible citizens. They will be the future decision makers and need to be able to judge on models by others, be it the credit rating in a bank or an expertise on relevant ecological issues. Starting at primary school with simple examples, they need to experience themselves the problems arising from translating situations of the world they live in into the disjoint world of mathematics. Heinrich Winter (1995) demands this as the first of his three indispensable basic experiences which pupils need to be exposed to in mathematics teaching.

I want to identify three important factors which can promote the development of modelling competence of pupils, but can also obstruct them drastically. These are

- The problem field "central examinations",
- The use of computers and
- The professional development and motivation of teachers.

This will be illustrated by concrete examples.

The first modelling competence that needs to be developed in school is an adequate handling of numbers, which we meet as the ideal numbers of mathematics, real numbers from the world we live in, and the often problematical computer numbers. Typical examples will show how the important competence to see the world with mathematical eyes can be developed. The acquired modelling competencies need then to be assessed by appropriate test phases.

Google's Page-rank-system – a present-day application of mathematics in classroom

Hans Humenberger

University of Vienna, Austria

Very many people use Google whenever they want to know something about a special word (item, term). When using Google one gets a list of sites containing the word one is looking for. In most cases the important sites (concerning the word we looked up) are more or less on the top, so it is not necessary to have a look on hundreds of sites to read something important and informative.

Here the following question arises naturally: *How* can Google manage this? How does Google know whether a special site is an important one and therefore should be presented quite on the top of the list?

It turns out that the answer is connected to the so called "Page-rank", a topic that can be dealt with in upper secondary school. Mathematically this is the field of matrices, vectors, solving linear equation systems, elementary stochastic processes (without needing the whole realm of the theories behind).

How can the process of using an internet search engine be modelled mathematically? This is not a task for students to work on for themselves (modelling processes with autonomous students' work) but it shows in a striking way how a few and easy assumptions can lead to a simple model and further on to really important results. The basic ideas behind the Page-rank-algorithm are quite simple and a basis for the founders of Google (L. Page and S. Brin). This does definitely not mean that "programming an internet search engine" is an easy task, quite on the contrary, but it can be seen as a confirmation that basic ideas still are very important and that one can establish really world shaking things like Google and make very much money by cleverly using both elementary and ingenious ideas.

Besides *modelling* another important didactical principle can be realized here in a very good way: Stressing and making visible the *connections* between several mathematical fields: One needs stochastics (probabilities), linear algebra (matrices, vectors) and analysis (limits) – these are the three main mathematics domains of upper secondary school.

In the talk I will present the basic ideas and concepts and show possibilities how teachers can deal with that interesting topic at school in a very elementary way.

The use of Mathematical Modelling in Understanding HIV/AIDS: A Case study of Sokoto State of Nigeria

Mohammed Olanrewaju Ibrahim, **Taiwo Hassan,**

Yomi Ogunyemi, Akor Franca Ene

University U.D.U. Sokoto, Nigeria

In this paper, computer simulation was carried out for three different mathematical models on HIV/AIDS, using the data collected from Usmanu Danfodiyo University Teaching Hospital, Sokoto in Sokoto state of Nigeria. Firstly, a brief description of the mathematical models their uses and assumptions made are presented.

A computer simulation was then carried out for the three different mathematical models of HIV/AIDS. A major problem with AIDS is the variable length of the incubation period from the time the patient is diagnosed as seropositive until he exhibits the symptoms of AIDS. This has major consequences for the spread of the virus. Hence, the first mathematical model dealth with the variable length of the incubation period.

The second and the third mathematical models were for the time evolution of the disease between those infected and those with AIDS. Using these models the estimated number of people living with HIV/AIDS in Sokoto state was predicted and also the implication of the simulation for short and long term of the cause of the epidemic was pointed out.

A Study on a Teaching Material Focused on Selecting Appropriate Data - Is Blue Really Advantageous in Judo? -

Sunao Ikahata

Michimori High School, Hukui, Japan

In Japan, the statistical learning contents in lower secondary school mathematics had disappeared with a reduction of the number of lessons in the last revision of the course of study (notified in 1998). However, the importance of thinking concerning statistics and probability in knowledge-based society has been realized again, therefore statistical learning contents in lower secondary school mathematics came back in this revision of the course of study (notified in 2008). And applying data is being emphasized with taking reflection on previous teaching biased toward knowledge.

Against the background, my interest here is to develop teaching materials that can foster students' practical thinking concerning statistics and probability.

Dijkstra & Preenen (2008) show that there is no effect of blue outfit (*judogi*) on winning contests in judo. In their study, some confounding factors in the analysis of Rowe *et al*. (2005, 2006) are pointed out: allocation biases, asymmetries in prior experience and difference in recovery time. The process of Dijkstra & Preenen's analysis of excluding inappropriate data is so interesting from the viewpoint of mathematics education.

In this paper, focusing on selecting appropriate data, I will attempt to change the analysis concerning effect of blue outfit in judo by Dijkstra & Preenen into a teaching material for lower secondary school students.

An historical perspective on how to make connections between modelling and constructing mathematical knowledge

Toshikazu Ikeda, Max Stephens

Yokohama National University, Japan;
University of Melbourne, Australia

One of the unresolved tensions in the teaching and learning of mathematics is how to strike a balance between modelling and pure mathematics. At the Rome 2008 Centennial of ICMI, Niss reiterated the importance of such a balance, challenging today's accepted opinion that instruction in mathematical ideas and techniques should come *first*, and *only then* might it be possible or even desirable for students to apply those ideas in some modelling activities. This study will look at a surprising resolution of that tension by examining Japanese textbooks for the junior and senior high school nearly seventy years ago – before and during the World War II.

At that time, the Ministry of Education required textbook writers and teachers of mathematics "to cultivate students' abilities to mathematize real world phenomena based on number, quantity and space, to treat the results mathematically, and to apply them to national life". Consequently, real world situations were incorporated into mathematical textbooks with a view to constructing mathematical knowledge. These situations had two distinct roles: first as objects to mathematize in order to solve real world problems; and second as evidence by which to test the *validity* of mathematical concepts. This is an interesting reversal of accepted opinion where mathematical ideas are simply assumed to possess *for students* their own validity.

As students' grasp of mathematical knowledge and techniques becomes more sophisticated, it is often thought desirable to seek out different contexts and situations to illustrate the usefulness of those ideas and techniques. By contrast, the Japanese textbook authors tended to use repeated instances of the same contexts through which new phases of mathematization could be developed. Thus, new and more sophisticated mathematical knowledge and techniques could be applied to familiar contexts and familiar results.

Several units were explicitly based on a series of real world questions. Sometimes, these sequences moved from specific real questions to discuss general principles; at other times they moved from the general to the specific; and explored connections, for example, between isotherms and isobars and the likely impact on wind direction.

While the context of early secondary education has changed greatly over the past seventy years, the guiding principles of curriculum design which informed these earlier textbooks continues to provide a helpful reference point – and a point of challenge to those assumptions which are too readily made today – in deciding how to balance modelling and the construction of mathematical knowledge in the teaching and learning of mathematics and in the writing of school textbooks.

Authentical modelling problems in mathematics education – examples and experiences from a modelling week

Gabriele Kaiser, Björn Schwarz, Nils Buchholtz

University of Hamburg, Germany

In March 2009 a modelling week took place at the University of Hamburg, during which over 350 students from 19 schools worked on complex and authentic modelling problems continuously for one week. The project was carried out as a joint activity of the department of education and the department of mathematics and was directed by professors from both departments. In addition on the part of the university parallel to the modelling week a seminar was carried out for future teachers who worked as tutors for the students. By this means the students were intensively supervised during their work on the modelling projects while on the other hand the future teachers had the opportunity both to gain practical experiences during the project and to reflect their work in the respective seminar.

The presentation will give a deeper insight into the work of the students during this project. In the first part of the presentation the structure and course of the modelling week will be presented in detail. Especially an overview of the broad variety of topics of the modelling problems worked on by the students will be given. Subsequently in the second part of the presentation one modelling problem will be described more in-depth. This problem covers the reproduction of ladybugs under the influence of a sexually transmitted disease. Starting point is the observation that, although large parts of the population of ladybugs are infected by this disease, the size of the population remains almost constant over a long period of time. The task for the students was to develop a mathematical model which reconstructs the development of the population size based on data given in an article.

The presentation will focus especially on authentic solutions brought out by the students who in the end effectively were able to develop a model which very well reproduces the data of the population of ladybugs. Furthermore the development of this model meaning the approaches of the students to get there will be part of the presentation.

Finally, experiences and results of an evaluation will be described, in which it became apparent, that the students were highly interested and motivated by these kinds of examples. Furthermore, they expressed their strong wish to include these kinds of examples in their ordinary lessons.

Enlarging the conceptual field of function by means of mathematical modelling: an investigation using conceptual maps

Lilian Akemi Kato, Silas Venâncio

Maringá State University, Brazil

This paper presents the results of a research about some of the potentials of mathematical modelling in the enlargement of conceptual field of function in students of the eighth grade of elementary school and the first grade of high school.

The chosen theme for the development of the activities was related to the type of fuel, alcohol or gasoline, which would be more advantageous in a bi-fuel car. In this research, the mathematical modelling was used as a prior organizer of students' preliminary knowledge to the basic ideas of the concept of function, besides enable them to develop significant relationships between their previous knowledge and the new situations which resulted from the activity.

It was adopted the construction of conceptual maps as the main strategy for identifying evidence of an enlargement of conceptual field of function, by the student. The assignment shows as a theoretical reference the Theory of Conceptual Fields by Vergnaud and it draws a parallel between the concepts of this theory and some of the epistemological features of Mathematical Modelling.

Accordingly, in the process of building the mathematical model, more emphasis was given to the discussions and action strategies presented by the students, which show an enlargement of their conceptual field.

The observations and analysis here performed, based on these theoretical references, showed the mathematical modelling as a strategy that led the students to the enlargement of the conceptual field of function.

The development of teaching materials using "Kepler's Law" for senior high school students who want to become scientists – with consciousness of the interrelation between math and science -

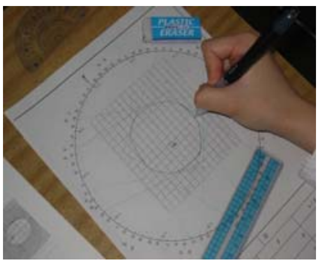
Tetsushi Kawaski , Seiji Moriya

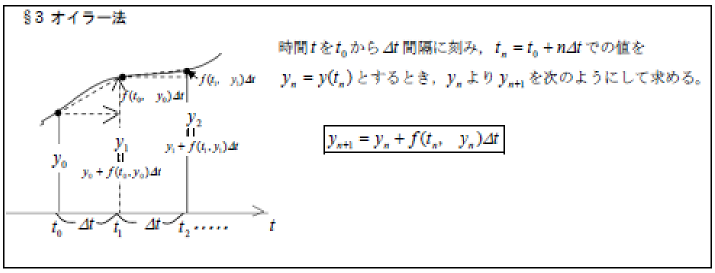
Kyoto Prefectural Sagano Senior High School,
Kyoto University of Education, Japan

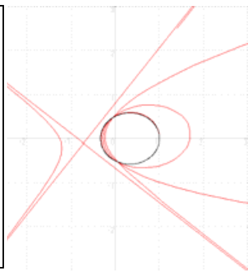
Abstract: The purpose of this study is to advance use of the differential equation in the field of calculus in future senior high school mathematics in Japan. Mathematics studied at high school must be applied to actual phenomena, and instruction which ties in to analysis is required. However, the present school environment regards itself as being outside of the mathematical domain on account of the small number of school hours. And, it is difficult to get the opportunity when the math teacher teaches science.Although the estrangement from science and mathematics is widely questioned in Japan, few people are aware of this as a directly contributing factor. In order to raise interest and concern in mathematical subject matter, it is necessary to connect mathematics to other fields of science and provide concrete examples. This will affect creativity of the technology in future Japan.

We have developed teaching materials using "Kepler's Law" for high school students who want to become scientists. For example, to draw figures of "Mercury Orbit" leads the students to better understanding and greater interest of the first law and the second law of Kepler's. The simulation of planetary movements increases their knowledge about the laws. Students realize the necessity of the differential equation. In addition, making the experiments and solving the problems contributed to fostering logical thinking.

Then, we made the curriculum from which the elliptic motion of the planet was gotten from the law of universal gravitation. By making the expression model and deriving the solution curve, students got great educative results. This publication suggests that the mathematics materials with physical points of view are effective for senior high school students.







The measurement of modelling competencies in students' assessment studies

Michael Kleine

University of Education Weingarten, Germany

The concept of competence holds a central position in comparative studies in mathematics. To show the results of these studies, the achievement is divided into intervals of competencies. It is a question of interpretation, however, whether this division corresponds with cognitive levels that are based on the empirical results of the studies. Up to now the description of these competence levels are insufficient with regard to students' modelling abilities.

In this contribution we take a look at mental models of mathematical concepts which are important for various aspects of mathematical thinking: "Grundvorstellungen", abbreviated to GVs. The results of a students' assessment study are presented, in which the modelling competencies with regard to GVs are the fundamental issue. The test instruments are created with a hierarchical design on the basis of a GV-classification towards modelling complexity.

The data are analysed with the method of Latent-Class-Analysis (LCA). On the base of this method it seems to be possible to describe competence levels against the background of the theoretical framework. These levels depend on qualitative classification of the criteria-based modelling abilities. The method (LCA) completes the quantitative measurement of the Rasch Model in the presented study. It will be explained how the use of LCA can be used as an example to outline the link between the constructive and empirical results of the modelling test.

Sense of Reality Through Mathematical Modeling

Jhony Alexander Villa–Ochoa, Carlos Mario Jaramillo López.

Universidad de Antioquia, Brazil

The purpose of this paper is to present the results of a research project approved by CODI[1], which deals with a qualitative case study of in the field of mathematical modeling. The study investigates the role of "real" life modeling situations of the learner in the construction of mathematical knowledge in classroom. The study of episodes, interviews, questionnaires and direct observations allowed analysis of how the teachers describe their teaching performance when approaching the content of school mathematics. However, the most important aspect was to be able to detect the necessity of a "sense of reality", which can be characterized as: the perception that a professor should have of his own reality and that of his students, which includes intuition and the ability to detect the situations and opportunities of the sociocultural context in which the knowledge of the students can be mobilized. This perception includes a good dose of imagination and creativity. The sense of reality, rather than a rational component of teacher knowledge, it is a subjective component, which in a metaphorical sense acts like a magnifying glass with which the teacher himself watches an objective reality and lets his students reinterpret this fact throug a process of mathematical modeling.

The development of this research shows that while there are teachers with a strong positive belief in the classroom and working with strong positive attitudes to change, it is necessary to develop a sense of reality as a methodology that could facilitate the interaction of relations between the sociocultural context and mathematics school, all through modeling.

References

Blum, W., Galbraith, P. L., Henn, H.-W., & Niss, M. (Eds.). (2007). *Modelling and aplications in mathematics education.The 14th ICMI Study.* New York: Springer.

Doerr, H. M., & Lesh, R. (2002). A modeling perspective on teacher development. In ,. R. Lesh, & H. M. Doerr (Eds.), *Beyond constructivism. Models and modeling perspectives en mathematics problem solving, learning, and teaching* (pp. 125-139). Mahwah: Lawrence Erlbaum Associates.

Doerr, H. M. (2006) .Teachers' ways of listening and responding to students' emerging mathematical models. *ZDM , 38 (3),* p.255-268.

[1]Research project in progress: The process of mathematical modeling in the classrooms of a school of Southwestern Antioquia. Acta CODI: 559, February 2008. Assigned to the Research Center of the Faculty of Natural Sciences. University of Antioquia (Colombia).

Modelling of infectious disease with biomathematics and bioinformatics: Implications for teaching and research

Ajit Narayanan, Sergiy Klymchuk,

Auckland University of Technology, New Zealand

Norbert Gruenwald, Gabriele Sauerbier

Wismar University, Germany

Tatyana Zverkova

Odessa National University, Ukraine

The paper compares a variety of models from biomathematics and bioinformatics of the spread of SARS (Severe Acute Respiratory Syndrome) that hit dozens of countries worldwide in 2003. All models were based on the real data for Hong Kong published by the World Health Organization (WHO). Although the models were based on the *same* data reported in March 2003, they gave *very different* predictions of the spread of the disease for 12 June, 2003 when the last case was reported in Hong Kong. The predictions varied from just under one thousand to over three million cases. Some models (linear, exponential and logistic) were built using differential equations and some using simulation programs from informatics. The models were discussed with first-year university students who were studying differential equations as part of their calculus course. A questionnaire was given to the students to find out their opinions on possible reasons of the differences in the predictions in the models, on their preferences to modelling either with mathematics or informatics and on the possibility of doing research projects on the topic. The students' responses are presented and analyzed in the paper.

Stochastic case problems for the secondary classroom

Usha Kotelawala

Graduate School of Education, Fordham University, USA

Engaging and motivating students is one of greatest challenges faced by secondary mathematics teachers. This presentation will provide a path to teaching important topics from probability through the use of stochastic methods in operations. Through real scenarios and visual modeling secondary students can consider the critical question, "what is the probability that the system will fail?"

The audience will be presented with problems at the secondary level to explore from a stochastic methods perspective. These problems will include scenarios such as,

- How many engines does a 747 need to fly? How many engines does it have? What's the probability of all engines failing at the same time?
- My old Christmas lights go dark when any one of them fails? What's it worth to pay for better bulbs?
- I've got to have music for Friday night. My three friends are both bringing their Ipods, but they complain that their older batteries give out often. Should I ask Talia to bring hers too?
- We used to have three copy machines on the 11th floor. But due to the drop in our endowment, they decided not to replace the one that failed last spring. Both remaining copiers still run into problems a couple times a month.

Examining these problems will be followed by a brief presentation of visual representations to assist student in understanding the varying possible scenarios and additional problems. The mathematics of these problems will be discussed for participants to consider how such problems could serve them in their classrooms.

In-service and prospective teachers' views about modelling tasks in the mathematics classroom – Results of a quantitative empirical study

Sebastian Kuntze

University of Education, Ludwigsburg, Germany

Professional knowledge and beliefs of mathematics teachers concerning the role of modelling for the mathematics classroom might have impacts on the way teachers conceive learning opportunities for their students. In particular, views about tasks requiring modelling steps might be relevant for instruction-related decisions of teachers with respect to modelling. However, there is still a need for quantitative empirical research about such views and corresponding professional knowledge. Consequently, this study concentrates on this area. Different levels of globality of teachers' views (Törner, 2002; Kuntze & Zöttl, 2008) were addressed: more than 75 in-service secondary teachers were asked about their instruction-related views, including views about modelling on a rather global level and asked to judge on characteristics of different tasks on a rather content-specific level. In order to find out whether these views are interdependent with a professional background rooted in classroom practice, 230 prospective teachers without long-term classroom experience were included in the study as a reference group.

The results indicate that these prospective teachers preferred tasks with rather low modelling relevance to those requiring more intensive modelling activities. These views might be linked to the fear of the prospective teachers that the goal of mathematical exactness might be underemphasised in modelling tasks. However, the in-service teachers' views differed significantly: the tasks requiring more intensive modelling activities were rated more positively.

These results indicate that the in-service teachers might have gained an increased awareness for the learning opportunities linked to modelling tasks on the base of their classroom experience.

However, on the level of the more global views on modelling, the in-service teachers felt, for example, on average rather unsure about their knowledge concerning the modelling cycle. These findings help to identify possibilities of fostering the teachers' professional knowledge. Moreover, the study highlights possibilities for deepening research on the structure of professional knowledge and instruction-related beliefs linked to modelling in the mathematics classroom.

References

Törner, G. (2002). Mathematical Beliefs – A Search for a Common Ground. In: G. Leder, E. Pehkonen, & G. Törner (Eds.). Beliefs: A Hidden Variable in Mathematics Education? (S. 73–94). Dordrecht: Kluwer.

Kuntze, S. & Zöttl, L. (2008). Überzeugungen von Lehramtsstudierenden zum Lernpotential von Aufgaben mit Modellierungsgehalt. Mathematica didactica, 31, 46-71.

Is there only one modelling competency?
The question of situated cognition when solving real world problems

Leiss, Dominik / Schukajlow, Stanislaw / Besser, Michael

University of Kassel, Germany

The debate about the concept of competencies has been the centre of the general-didactical and the subject-specific-didactical discussion in the last decade (cf. among others McLaughlin et al. 1995, Niss 2003). The great importance of the concept of competencies within the research on teaching and learning is reflected in the mathematics education discourse, i.a. in the central role of competencies of educational standards in mathematics (NCTM 2000, KMK 2004, Oelkers & Reusser 2008). One of the central competencies within the German educational standards in mathematics is the competency of "mathematical modelling" that can mainly be described by the ability of successful translating processes between reality and mathematics (cf. Leiss & Blum 2006, Greer & Verschaffel 2007).

The problem of situated cognition (cf. Brown & Collins 1989) constitutes an open question also in the research of the modelling competency. While the problem of transfer of acquired (modelling) competencies from classrooms into the real life has been diagnosed as an important topic within the current mathematics education research (cf. amongst others Niss, Blum & Galbraith 2007), it seems to be unexplained to a large extent if the acquired modelling competency that was learned within a particular mathematical school context (e.g. "Pythagoras' theorem") can be transferred without great effort to other central parts of mathematical instructions (e.g. "linear functions"). Correspondingly, the in hand study centres the following questions:

1. Is it possible to identify pupils' different kinds of strengths and weaknesses regarding various contents and competencies?
2. Based on this aspect: Is it possible to infer various modelling competencies within different contents in a qualitative and quantitative way?
3. What consequences can be inferred from these observations for the practical mediation of competencies?

In order to be able to answer these questions, we will hark back to empirical data (tests, school tasks, videos, questionnaires etc.) from the following two interdisciplinary research projects DISUM (project leader: W. Blum/ R. Messner/ P. Pekrum) and COCA (project leader: E. Klieme). Among other aspects, studies relating to modelling competencies of pupils who are in grade 9 take centre stage in both projects. In addition to quantitative/ psychometric results, qualitative results that make clear how far actual modelling processes vary in specific topics will be considered in this lecture.

Design Research in Mathematics Education
- Workshop for Early Career Researchers -

Richard Lesh

Indiana University, Bloomington, USA

Design research is especially relevant to research on teaching, learning, and problem solving that emphasizes the development of future-oriented curriculum materials; and, in particular, it is especially relevant to teaching, learning, and problem solving that focuses on modeling and applications. Why? This will be explained in the session. … After a brief introduction describing the fundamentals of "design research" in STEM education, this session will aim at helping participants develop a one-page prospectus describing of a *design research project* which might complement or replace more traditional investigations that they had in mind. The session will be based on recent *Handbook of Design Research in STEM Education* (Kelly, Lesh & Baec) – which in turn was based on a large project funded by the USA's National Science Foundation and involving leading researchers across a wide range of topics in science and mathematics education.

In mathematics education, design research methodologies largely evolved out of "teaching experiments" that were emphasized in the 1980's and 1990's. But, they also borrowed heavily from methodologies that have long histories of being used in "design sciences" such as engineering – where, like in math/science education, many of the most important "things" that need to be understood and explained are being designed or developed by the same research communities that investigate them. So, design scientists are continually changing the "subjects" they are studying – and projecting their conceptual systems into the world in the form of designed artifacts and tools. So, as soon as they understand these systems, or artifacts or tools, they tend to change them. And, this implies that every explanatory systems and every product that these explanatory systems are used to design or development tend to be the n^{th} in continuing sequences of design or development.

By borrowing and adapting methodologies from design scientists, the intent was increase both the practical importance and the theoretical cumulativeness of STEM research. In particular, leading STEM researchers were especially interested in developing alternatives to naïve methodologies that have been dubbed to be a "gold standard" for research in education – in spite of the fact that such methodologies tend to be based on assumptions which are significantly out-of-step with modern conceptions of most of the "subjects" that are priorities for math/science educators to understand.

Do Inservice Teachers accomplish Mathematical Modeling the same way Students do?

Thomas Lingefjärd

University of Gothenburg, Sweden

To select mathematical modeling tasks for teaching is a delicate matter. In general, mathematical modeling tasks are more complicated than more routine oriented tasks and demands much more from the person who is trying to solve the modeling problem. In general, mathematical modeling tasks ask for mathematical knowledge, technical expertise, conjectures, argumentation, conclusions and a good writing.

Teachers of mathematical modeling at all levels must consider many estimated consequences when suggesting mathematical modeling assignments. Is the selected task to difficult? Alternatively, is the selected task to easy? What will the estimated time for solving the task be? How complicated will the handle of technology be, both in terms of maneuvering the tool and in terms of analyzing the results? Are there concepts in the mathematical modeling process that will be alien and unknown, thereby causing confusion and disturbing the mathematical modeling process?

With my background of having taught mathematical modeling to prospective teacher for many years, and during some years also to prospective engineers, it is sometimes hard to identify ideal or at least suitable mathematical modeling tasks for students in upper secondary school. The work in the Comenius Network "Developing Quality in Mathematics Education II" has the main focus of developing and evaluation of mathematical modelling tasks. In general we do this through mutual exchange between teachers and researchers across eleven European countries, complemented by local discussions in each country.

During the fall last semester of 2008, I was asked to teach mathematical modelling to a group of Swedish upper secondary teachers. They were all experienced and well educated teachers, but no one of them had any course of mathematical modelling in their teacher training. So how do one plan and implement a course in mathematical modelling for teachers?

As for take home assignments, I decided to give three different problems, one from the medicine project in the DQME II project and one that I gave my students in the teacher training program at the University of Gothenburg several years ago. The problem is described in the conference book from ICTMA 10. But I also decided to give a very open mathematical modelling situation, such an open problem that it probably would be impossible to give it to students.

In my talk, I will share the reactions and opinions I received from the experienced teachers when they were put in a situation very similar to what their students sometimes experience when facing open assignments.

Sunrise in Sweden and Germany

Thomas Lingefjärd & Stefanie Meier

University of Gothenburg, Sweden
Technische Universität Dortmund, Germany

The work in the Comenius Network "Developing Quality in Mathematics Education II" has one main focus on the development and evaluation of modelling tasks. This is currently taking place via mutual exchange between teachers and researchers.

During the project meeting in Cluj Napoca, Romania (September 2008), it was decided that several of the eleven countries in DQME II wanted to work with the same modelling problem: The sun hour project. All together Denmark, Germany, Hungary, Italy, Poland and Sweden decided to test the sun hour project in their different schools. At the beginning of 2009, one class in Sweden and one class in Germany worked on the sun hour project. In short, the task was to ask the students to develop a suitable mathematical model to describe the phenomenon sunrise/sunset and the change of the daylight time (number of sun hours) for their home town. The model should be able to account for differences in location around the globe. The worksheet for the students had been created in mutual exchange between the teacher in Germany and the teacher in Sweden.

In Sweden, the teacher gave his students (age 18) the problem one week ahead and then he asked them to be prepared to present a mathematical model which could be used to predict the number of sun hours at a different location. The students were not given any coaching or support from the teacher during that week. They all worked in groups with 3 or 4 students in every group, and they were informed that their presentations could affect their grade on the course they were studying (Mathematics D in the Swedish gymnasium). Important concepts like longitude, latitude, the Earths rotation axis, vernal equinox, summer solstice, autumnal equinox, and winter solstice were discovered and investigated along the way. Some of the students discovered that trigonometric curves are natural when you urge to describe the fluctuation in sun hours during the year.

In Germany the teacher gave his students (age 17-18) the task before the Christmas holidays so that they had two lessons to prepare the group work for the project day in January. The groups decided on which element they want to focus. During that one day they had four lessons to search information in the internet and collect this information, with the teacher being present to support them. They not only tried to understand the mathematics in this information, but also managed to understand the connection between the real situation "sun hours" and the mathematics. During the two lessons of presentations each piece was helpful to understand the whole context better. At the end of this project day the students had an idea of a suitable mathematical model to describe the phenomenon sun hours at different locations.

The presentation of this paper will focus on the differences in handling this task for teacher and students purposes in both countries.

Blockages and barriers in students' work on modelling tasks

Matthias Ludwig*, Binyan Xu±

PH Weingarten, Germany, ECNU Shanghai+, China*

The focus on research on teaching mathematical modelling both in Germany and China is on the increase of the success of students by solving modelling tasks. In this context there are many qualitative studies who discuss mostly single solutions of students. But there are only a few empirical large scale studies (DISUM-Study, Blum, Leiss) in this context. In this empirical comparative study (n>1100) we explore the students' solutions on a modelling task which is based on a real situation (peeling a pineapple). We are interested which levels the students from grade 9 (15 years old) to grade 11 (17 years old) from China (n>600) and Germany (n>400) will reach by solving this modelling task. To define the different levels (level 0 to level 5) we have a look on the transitions in the modelling cycle from Blum and Leiß (Blum & Leiß, 2005) in a similar way Galbraith and Stillman (Galbraith and Stillman, 2006) have done. For each level we define some possible parts of the whole solution. With these definitions we could allocate each student to one single level from 0 to 5.

Our research questions for this study are for example: Can we find some differences in the performances of the Chinese and German students? Are there some gender differences? Is there a trend in the performance over the grades? Do we find different blockages and barriers between German and Chinese students?

With statistical methods we could show, that there is no statistically significant difference in the performance between the German and Chinese students. But it is very interesting, that there are different levels of barriers and blockages among the two populations. We found, that for Chinese students level 2 is a barrier (this means that level 3 or more is reached by less than 50% of the students who reached level 2), but for German students level 4 is the barriers. For this you have to know what level 3 means, that the student can work in the mathematical model found by himself. Level 4 implied in addition that the students can reason and finish their calculations. We also found that performance became better when the students are older and we found that in China in every grade the girls performed better than the boys. In Germany the girls performed better in grade 9 and 10 but not in grade 11. It is remarkable, that no German girl achieved level 5.

References:

Blum, W. & Leiss, D. (2005). How do students and teachers deal with mathematical modelling problems? The example "Sugarloaf". In: ICTMA 12 Proceedings, 222-231

Galbraith, P., Stillman, G. (2006). A Framework for Identifying Student Blockages during Transitions in the Modelling Process . ZDM 2006 38(2), 143-162

Professional development about modelling within a European context Results of a European project

Katja Maass, Johannes Gurlitt

University of Education, Freiburg, Germany

Across Europe there is increasing awareness that school pupils need to have more experience of applying mathematics in such a way that they might become well informed and actively critical citizens and workers. This requires new competencies of teachers who, in the main, do not, at present, successfully integrate applications of mathematics into their daily classroom practice.

Within the European project LEMA (Learning and education in and through modelling) partners out of six countries developed a five-day professional development intervention.

The development of the course was informed by the theoretical background about modelling professional development as well as an analysis of need. Whilst promoting a common approach, the course is flexible and adaptable to the requirements of partnership nations and others beyond. Target groups area teachers in primary and lower secondary phases, offering both initial and in-service training.

To evaluate the impact of the course a questionnaire was designed. The questionnaire was prepared to assess beneficial effects of the training on teachers taking part in the course (6 countries). Items were designed to mirror the theoretical background and key-aspects of the modules of the professional development. Thus the questionnaire included the knowledge, beliefs and the self-efficacy to implement mathematical modelling in classroom practice.

To measure possible knowledge gains and belief-changes we implemented a pre-post-control-group design. Results show medium to large effects for the training group compared to the control group for knowledge about modelling and it's application as well as self-efficacy. Beliefs about mathematical modelling did not change. Acceptance ratings oft he training (e.g., support, structure, communication, atmosphere, etc.) were high.

The paper presents the project itself, the professional development intervention and the results of the evaluation across all nations.

Comparing Modelling by Graduate School Students and by Electronics Expert in Solving Electronic Problems: Through Visualizing Modelling Progress by Using the Method of Applied Task Analysis Mapping

Akio Matsuzaki

Naruto University of Education, Japan

Modelling progress is different for each modeller because their experiences are different, and changing models are influenced by some situations. So I propose to describe modelling progress by using a method based on applied task analysis mapping and situations. This method is reflecting situations of each modeller and is able to visualize modelling progress. In this paper the keyword *situations* is used in its broad meaning, i.e. it comprises images based on each modellers' experiences. Furthermore with this concept it is possible to visualize shift of modelling phases and connection between variables that construct models.

I will introduce two modelling problems related with solving electronics problems. The problem is 'How much brightness is needed to read a book?' I asked the test persons the following procedures to answer the problems: to decide conditions, pose models and solve problem. Additionally I asked them to talk about situations by think-aloud methods through solving the problems. Until now I have already researched different working groups; for example, paired learning by upper secondary school students. In this paper I will compare two different test persons; one person is a graduate school student who has studied physical science education, and another one is an electronics expert who works with electronics job.

In the case of the graduate school student he applied physical science knowledge and skills in modelling; for example, diffusion of light or relationships between distance and brightness. And he imaged some situations based on his experiences; for example, lighting instrument in operating rooms. On the other hand in the case of electronic experts he applied his expertise of electronics job; for example, standards of brightness or wavelength of fluorescence. He described in the questionnaire that he used these features in his job, and he tried to solve his own posed problem with statistical knowledge and skills. Modelling progress is described by using the method of applied task analysis mapping, which shows the influences by test person's own situations. The situations include not only mathematical situations but also non-mathematical situations. Non-mathematical situations indicate learning experiences or job experiences, and these situations are variables in constructed models in modelling. For chasing real modelling progress it is necessary to use the keyword *situations* in its broad meaning including variables constructed models.

Spatial logic for solving geographical problems

Marco Meyer, **Engelbert Niehaus**

University of Koblenz-Landau, Landau, Germany

Spatial problems, taken out of the area of geography, offer a lot of aspects for mathematical modelling, which need the field of geometry, algebra, statistics and analysis for the solving. An option to solve these problems are the geographical information systems, which are computerised systems, which consists of hardware, software, data and applications. With the geographical information system data can be digitally recorded, saved, reorganised and spatial analysed, as well as exported and graphically illustrated in different types. This data can occur as point information or as areal information. If we handle with areal data, it is no problem to derive point information for locations in these areas from the data stored for the whole area. But most of the time, we have to deal with point information, for example measured exposure rates at special location with the coordinates *(x,y)*, so we can not make any statements of the areas around that special location.

In Boolean logic, we can assign *1* to statements which are true, and *0* to statements which are false. In the upper mentioned case of measured exposure rates, we can assign *1* to the *(x,y)* coordinates if we measured a value above a critical value, or *0* if we are below. This leads to a function:

$$f : \mathbb{R}^2 \to \mathbb{R} \qquad (x,y) \to f(x,y) = z \in \{0,1\}$$

But this is no help for areas around that location, because this model cannot make any statements around the measure points. By interpolating the areas between the measure points, we receive a risk map, but with gradual transitions between 0 and 1.

$$\mu : \mathbb{R}^2 \to \mathbb{R} \qquad (x,y) \to \mu(x,y) = z \in [0,1]$$

Therefore we extend the Boolean logic to fuzzy logic, which now allows us to interpret those gradual transitions as a gradual membership to the fuzzy set "high exposure rate". So now we have the opportunity to make a statement for every point *(x,y)*. By adding more linguistic values, each represented by a membership function, we can model the OR and AND operator and for each function the NOT operator and derive fuzzy logical conclusions from this by a fuzzy rule system.

These membership functions can be plotted with the help of a computer algebra system, such as maxima. With these visualisations, pupils are able to understand the meaning of these risk maps, because they know 3d-maps of landscapes, so the higher the peak, the higher is the risk. They are able to plan routes through non risk areas for example, just by looking on the visualisation, without the need to know, what exactly has been done mathematically in the background.

Integrating Engineering Model Eliciting Activities in Elementary School Mathematics Curricula

Nicholas G. Mousoulides* & Lyn D. English**

**Cyprus University of Technology, Cyprus*
***Queensland University of Technology, Australia*

Providing engineering experiences to elementary school students is an increasingly important domain of research by mathematics, science, technology, and engineering educators. Recent research has raised questions about the context of engineering problems that are meaningful and engaging for young students. This paper will provide one response to such questions, namely, implementing engineering model-eliciting activities as a powerful way of promoting engineering and modeling within the mathematics curriculum (Zawojewski, Diefes-Dux, & Bowman 2008).

Engineering model-eliciting activities present students with real-world engineering situations in which students repeatedly express, test, and refine or revise their current ways of thinking as they endeavour to create a structurally significant product—namely, a model that can be used to interpret, explain, and predict the behaviour of one or more systems defined by the problem. For example, engaging students in hands-on, real-world engineering experiences involves them in design process cycles that utilize powerful mathematical problem solving and reasoning processes.

An environmental engineering problem, namely, the *Natural Gas Activity,* is presented in the paper. Given the worldwide reserves of natural gas in 1993 and the yearly average consumption, students were asked to calculate when the reserves of natural gas will be exhausted. Six groups of 12 year-old students worked on the activity for around ninety minutes. During their explorations students could freely use the Web for accessing various resources. There were a range of models that adequately solved the problem although not all models took into account all of the data provided. The models varied in the number of problem factors taken into consideration and also in the different approaches adopted in dealing with the problem factors. A number of students experienced difficulties in fully understanding and using the concept of *average* in developing their models. Two groups of students developed more coherent models, by incorporating current reserves, and how the use of renewable energy will affect the consumption of natural gas.

The study will conclude with arguments for integrating engineering model-eliciting activities in the elementary mathematics curriculum. These activities provide rich opportunities for students to deal with engineering contexts and to apply their learning in mathematics and science to solving real-world engineering problems.

References

Zawojewski, J., Diefes-Dux, H. A., & Bowman, K. (2008). Models and Modeling in Engineering Education: Designing Experiences for all Students. Netherlands: Sense Publishers.

Modelling with sketchpad to enrich students' concept image of the derivative

Mdutshekelwa Ndlovu, **Dirk CJ Wessels and Michael D Devilliers**

Tshwane University of Technology, University of South Africa, University of KwaZulu-Natal, South Africa

In the paper we discuss the pedagogical role that the dynamic mathematics properties of *Geometer's Sketchpad* can play in modelling and simulating the derivative and related notions in introductory calculus within a hypothesized learning trajectory. The proliferation of information and communication technologies challenges mathematics educators to engage in experimental research in the wide range of possibilities of enhancing the teaching and learning of mathematics in general and calculus, in particular. In this connection Cuoco (2002) notes that the proliferation opens up a whole new set of mathematical possibilities for students and educators more so when a new tool is designed to serve one field (mathematics) but is used in another (mathematics education).

The hypothesized trajectory was inspired and informed by the historical development of the derivative as gleaned from the literature on the history of the calculus. The models and modelling approach proposed by Lesh and Doerr (2000) influenced the choice and sequencing of model-eliciting activities to develop students' qualitative understanding of the concept. In an empirical study carried out by the first author (Ndlovu, 2008) involving first year non-mathematics major undergraduate science students at a university in Zimbabwe, six forms of representation enabled by *Sketchpad* were identified. In this paper we describe the sequence of *Sketchpad* activities, the nature of the six representations and possible ways in which students can translate from one form of representation to the other. The study suggests that multiple representations can enhance mathematizing and conceptual understanding in learners. The study also reaffirms that the pedagogical value of technology does not lie in the technology itself but more critically in the ingenuity and innovativeness of the teacher in instrumenting it.

References

Cuoco, A. (2002). Thoughts on reading Artigue's Learning mathematics in a CAS environment". *International Journal of Computers in Mathematical Learning* 7: 293-299.

Lesh, R. & Doerr, H. (2000). Symbolizing, communication and mathematizing: key components of models and modelling. In P. Cobb, E. Yackel, & K. McClain, (eds), *Symbolizing and communicating in mathematics classrooms* (pp. 361 -383). Lawrence Erlbaum Associates: Mahwah, NJ.

Ndlovu M. (2008). *Modelling with Sketchpad to enrich students' concept image of the derivative: developing domain specific understanding.*Unpublished DEd thesis: University of South Africa.

Improving learning in science and mathematics with exploratory and interactive computational modelling

Rui Gomes Neves , Jorge Carvalho Silva and Vítor Duarte Teodoro

Universidade Nova de Lisboa, Portugal

Research in science and mathematics involves a balance between theory, experiment and computation. In an approach to science and mathematics education in phase with today's rapid scientific and technological development, an early integration of scientific computation in the learning environment is of crucial importance. On the other hand, the learning environment itself should reflect the exploratory and interactive nature of modern scientific research.

However, computational methods and tools, as well as interactive and exploratory learning environments, are still far from being appropriately integrated in the high school and university curricula in science and mathematics. As a consequence, these curricula are generally outdated and most tend to transmit to students a sense of detachment from the real world. These are contributing factors to the development of negative views about science and mathematics education, leading to an increase in student failure.

In this work, we discuss how Modellus, a freely available software tool (created in Java and available for all operating systems) can be used to develop computational modelling learning activities in science and mathematics with an exploratory and interactive character. These activities can be adopted by high school and university curricula. They may also be a valuable tool for the professional development of high school teachers. Focusing in physics and considering mathematics in the context of physical problems, we describe a selection of workshop activities on introductory mechanics which were implemented in a new course component of the general physics course taken by first year biomedical engineering students at the Faculty of Sciences and Technology of the New Lisbon University. The activities were designed to emphasise cognitive conflicts in the understanding of physical concepts, the manipulation of multiple representations of mathematical models and the interplay between the analytical and numerical approaches applied to solve problems in physics and mathematics. In this work we discuss the educational goals and the impact of the activities on the students learning of key physical and mathematical concepts in mechanics.

Mathematical Application during an Interdisciplinary Project

Dawn Kit Ee Ng

National Institute of Education,
Nanyang Technological University, Singapore

Interdisciplinary activities involve contextualised tasks claimed to promote a student-centred approach to learning and the use of higher-order thinking skills. They supposedly bring forth *connections* between real-world concerns and school-based knowledge as well as highlight links between various disciplines of knowledge for learning and problem solving. Research suggests that mathematical tasks situated within real-world constraints provide meaningful contexts for problem solving experiences and mathematical learning. However, there are challenges to ensuring the production of *quality mathematical outcomes* during contextualised tasks.

A design-based interdisciplinary project with mathematics, science and geography as anchor subjects was implemented in a total of 16 classes of students (N = 617) from grades 7 and 8 (aged 13-14) in two educational streams across three Singapore government secondary schools. The project, conducted through weekly meeting sessions over a 15-week period, followed the theme of "environmental conservation" where students worked in groups of four to design an environmentally friendly building at a location of their choice within Singapore. Each student-group was also expected to construct a physical scale model of their building from recycled materials. Mathematical tasks with written components in the project included: (a) scale drawings of the actual building and (b) cost of furnishing and fitting out a chosen area in the building.

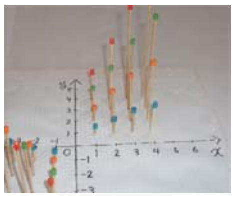
The research tracked the nature of collaborative mathematical knowledge application during these two tasks of the interdisciplinary project where 10 case-study student-groups mathematised the requirements of the project. Average-stream groups displayed higher amounts of mathematical engagement with the costing task than their high-stream peers but this did not translate to higher levels of complexity in the mathematics applied nor better quality work. The types and complexities of mathematics applied did not appear to be stream dependent. Some students had difficulties with scaling and spatial visualisation and there was limited activation and application of real-world knowledge. Some groups engaged in more fruitful knowledge application processes than others because they embarked on interpersonal monitoring of mathematical thinking and had good social relationship among members.

Research into teaching multi-variable functions – modelling, partial differentiation and double integration

Yoshiki Nisawa, Seiji Moriya

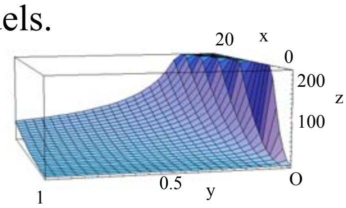
Kyoto Prefectural Rakuhoku Senior High School, Japan
Kyoto University of Education, Japan

In Japan, the teaching of mathematical functions to high school students treats only one variable. However, many of the phenomena in their lives are expressed mathematically by two or more variables. Therefore, it is very important for them to study functions with several variables. I would like to propose a method for teaching multi-variable functions.



First of all, I give students an easy mathematical expression to model, and have them analyze the range of the function with respect to the variables. They make a 3D model to deepen their understanding of mathematical functions and models.

Next, I give them a specific example to model, either the braking distance of a car or the propagation of sound. These expressions contain the quadratic functions, the fractional functions, and the trigonometric functions studied in high school. They study these and begin to look for other examples of these models.

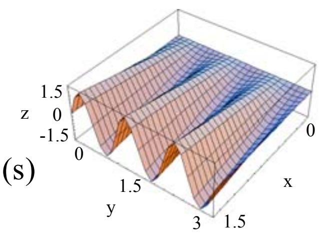


3D graph of (1)

Breaking Distance Example:

m = mass (kg); v = velocity, x (m/s); μ = a coefficient of friction, y;
g = gravitational acceleration $\left(9.8\,\mathrm{m/s^2}\right)$; d = stopping distance (m)

$$\frac{1}{2}mv^2 - \mu mgd = 0 \iff d = \frac{v^2}{2g\mu} \iff (1)\ f(x,y) = \frac{x^2}{19.6y}$$



3D graph of (2) (t=1)

Sound Propagation Example:

A = amplitude of wave motion, x (dB); f = frequency, y (Hz); t = time (s)

$$y = A\sin 2\pi ft \iff (2)\ f(x,y) = x\sin 2\pi ty$$

Finally, the students study partial differentiation and double integration. This is not currently in the curriculum for high school students. However, I teach this to the university student beforehand who hope to become mathematics teachers in high schools. I want to propose a better curriculum for high school student based on this.

From this teaching material, high school students will learn to positively construct mathematical models throughout their lives and gain the ability to analyze many situations.

Mathematical Modelling in Secondary Education. A case study

José Ortiz, **Aldora Dos Santos**

Bolivarian School, "Monseñor Francisco Miguel Seijas", Tinaquillo, Estado Cojedes, Venezuela

University of Carabobo, Campus la Morita, Maracay, Estado Aragua. Venezuela

Mathematical modelling in secondary education promotes the students to recognize the importance of the mathematics to understand contexts of local or global interest. In this study, we are interested in solving problems from the physical and social world. The participants are a group of five students of the first year of the secondary school (11 to 13 years old) located in an urban zone of Venezuela. The study contemplates to the followed processes and the representations used in the problem solving. This research is a work of field framed in a case study under a qualitative approach. It was applied a questionnaire with five problems and an interview. The findings reveal that the modelling schemes that were demonstrated in this study are framed in their majority in present proposals of mathematical modelling, when identifying the problem situation and interpreting the solution found under a context of the real world. With respect to the representation systems, the students go to the structuring of numerical answers with measure units and verbal representations. There is an absence of graphical representations. The students would need to receive a mathematical education with more connections between real world and mathematical world, which emphasizes more strongly the use of different representations systems.

Mathematical modelling of curved structures

Juha Paavola, Eero-Matti Salonen

Professor of Structural Mechanics, Finland

Traditionally in textbooks of mechanics, the derivation of various basic equations is performed by applying the tools of differential geometry. The differential geometry serves a way to draw simple figures of a deforming element of a body, and build up expressions by considering changes between the deformed and original configurations. Similarly, the figure showing the free body diagram of an element is usually used when deriving equilibrium equations. These figures are very graphic and handy, and easy to utilise in teaching, when the geometry of the body considered is simple enough. But, any double curved geometry, and particularly nonlinear analysis produce figures impossible to handle in this way. These difficulties will be bypassed by looking for other suitable tools from basic mathematics and mechanics.

The idea of this talk is simple – to develop generic mathematical tools which could be applied in a similar way with any structural model. The procedure can be pictured as a mill in which the geometry and the kinematics of a model are thrown in, yielding after certain grinding the final results – basic equations needed, like expressions for strains, equilibrium equations etc. This mill is built up by utilizing tools of numerical computation technique, combined with familiar vector calculus. Another important aspect is to create analysing methods for teaching, in which only basic mathematical tools – in this case the vector calculus - will be needed, familiar to most of students.

The procedure consists of the presentation of the geometry and displacement of a body by vector fields. These will be incorporated in the definitions of various physical quantities, defined also by vector calculus. The difficulties due to the complicated geometry will be by past by utilization of a local Cartesian frame, the invariance of the strain energy and the principle of virtual work. A local Cartesian frame was originally introduced by B. Irons when developing various shell theories for numerical purposes. When a local Cartesian frame is established in a clever way, the basic kinematical expressions can be obtained for any arbitrary curvilinear geometry without the doubt of the figures of differential geometry. The situation is exactly similar with the equilibrium equations and the principle of virtual work.

Choosing the kinematics is a basic task in structural modelling – needing continuous emphasising in teaching. Usually, it will not be explained explicitly, that the difference between various beam, plate and shell theories is hiding in the kinematics adopted. It is also worth noticing that it is a tool controlled by the analyzer himself. The geometry, loading, the accuracy looked for, are the factors to be taken into account, when choosing proper kinematics. The same ideas give the starting point for both analytical and numerical considerations.

Modelling and the Academic Characteristics of Applied Mathematics

Jacob Perrenet, Hennie ter Morsche

Eindhoven University of Technology, Netherlands

The Bachelor curriculum of Applied Mathematics in Eindhoven includes the so-called Modelling Track: a series of modelling projects where pairs of students solve mathematical problems posed in non-mathematical language. Training in communication and academic reflection are integrated in this track. See Perrenet and ter Morsche (2004) for a detailed description of the Modelling Track.

In the Netherlands, a new system of accreditation of higher education programmes has been introduced. Universities have to prove their academic merits with their programme content characteristics. The Eindhoven University of Technology has taken an active part in this process, especially concerning the scientific technological aspect. A set of criteria has been developed that consists of seven competency domains and four dimensions of academic acting and thinking. The university graduate should be competent (1) in one or more scientific disciplines (2) in doing research, (3) in designing, and (4) in co-operation and communication; he or she should (5) have a scientific approach, (6) possess basic intellectual skills, and (7) take into account the temporal and social context. The graduate's thinking and acting is characterized by (a) analyzing, (b) synthesizing, (c) abstracting, and (d) concretizing (Meijers, van Overveld and Perrenet, 2005).

With this framework the university has started to describe, analyse and test its programmes. The programme board of Applied Mathematics was one of the earliest to volunteer for an extensive series of interviews with all its staff members about their educational intentions. Data analysis resulted in the academic profile of the programme. This paper will consider the results on the following question:

Are their certain academic characteristics typical for the Modelling Track compared to the characteristics of the other subjects in the Eindhoven Bachelor programme of Applied Mathematics?

We will reflect on the importance of the Modelling Track considering the academic merits of the Eindhoven mathematics programme: is it fringe or backbone, or something in between?

Perrenet, J. & Morsche, H. ter (2004). Modelling as a Foundation for Academic Reflection in the Mathematics Curriculum; in ICMI Study 14: Applications and Modelling in Mathematics Education, Pre-Conference Volume of the Study Conference in Dortmund, February 2004; Universität Dortmund, pp. 211-217.

Meijers, A.W.M., Overveld, C.W.A.M. van & Perrenet, J.C. (2005). Criteria for Academic Bachelor's and Master's Curricula; Eindhoven University of Technology; http://w3.ieis.tue.nl/en/subdepartments/av/platform_academic_education/.

Pre-service teachers modelling annuity-based scenarios

Craig Pournara

Wits University, South Africa

There are essentially two methods to determine the balance in an annuity-based investment at any point in time. The first is an iterative calculation that focuses on the account balance. In this case attention is paid to the previous balance, the new deposit, and the interest on the total amount. This is referred to as the *account balance approach*. The second method focuses on the behaviour of each deposit over time. Here one is concerned with the ways in which each payment and it's associated interest combine to produce the account balance. This is referred to as the *individual payment approach*.

In South Africa when students are introduced to annuities in secondary school mathematics or in university mathematics courses, they are generally directed very quickly to the individual payment approach, where the focus on individual payments leads to a geometric progression, and ultimately to the formulae for future value and present value of an annuity.

In a course on introductory financial mathematics for a group of 40 pre-service secondary mathematics teachers, student teachers were required to develop a model for an annuity-based scenario. Data were collected from this activity as well as an activity involving missed payments. The findings suggest that the individual payment approach is not an intuitive model for students, but that most choose an account balance approach at first. Furthermore, several students did not readily accept the validity of the individual payment approach after being introduced to it. However, evidence from written work and interviews suggests that once students are convinced of the validity of the individual payment approach, it becomes a robust model for them which they are able to manipulate to deal with annuity-type scenarios that depart from a perfect payment plan. The use of spreadsheets that foreground the structure of the individual payment approach appear to support students in developing a robust understanding of the structure of annuities.

In the context of mathematics teacher education, these findings are important because they point to the ways in which school learners are likely to model annuity-based scenarios when they are not given a procedure to follow. In addition, it is important for teachers to develop strategies to help learners shift between account balance and individual payment approaches in solving annuity-based problems.

Using Mathematical Modelling to allow students to make their thinking visible.

Mr Trevor Redmond, **Ms Joanne Sheehy**

A.B. Paterson College, Gold Coast Australia

In many classes students of mathematics ask the question "Why do we have to learn this?" Their view of mathematics is one of working out of a text book where the answers are provided in the back of the book. Many students do not see the relevance of the mathematics that is developed and used in this way. Providing authentic contexts that have a real life focus is one way that allows students to see the relevance of the mathematics they are learning and mathematical modelling provides the tools that allow students to be able to interact with that authentic context. Through the modelling process students are afforded the opportunity to make thinking visible.

The Queensland Studies Authority in Queensland, Australia has determined that Modelling and Problem Solving will have a significant focus in its current Mathematics B syllabus. As a result, Queensland schools who offer this syllabus to their students need to provide evidence of students engaging in the mathematical modelling process.

The school used in this study had established a mathematics programme from Year 6 through to Year 12 that had as its major focus, mathematical modelling. Based on the work of David Perkins and the teaching pedagogy, "Teaching for Understanding", the mathematics department of this College had identified a sequence of generative topics across the year levels that are used to build the mathematical understandings required by the students. The mathematical concepts that related to the generative topics were identified and authentic tasks were developed. Through a study of the mathematical concepts, the students were taught to use the tools of mathematical modelling, as they related to the specific mathematical concepts under investigation throughout the topic, to explore the tasks.

This paper will investigate how the students were able to use mathematical modelling to make their thinking visible. The paper will consider a number of learning tasks and subsequent assessment tasks including the generative topic's culminating performance which asked students to investigate Newton's Law of Cooling. These tasks were situated in a Year 12 generative topic. The culminating performance instructed students to consider two data sets, the first data set had to collected by the students and satisfy a set of restrictions while the second data set was of the students' choosing, but again had to satisfy a set of restrictions. From these two specific instances, students were asked to develop a generalisation.

Mathematics & Real Life:
A new approach to teaching and learning mathematics

Anna Salvadori, Primo Brandi

Dipartimento di Matematica e Informatica – Università degli Studi di Perugia

In the last years, the requirement of innovative mathematical education has increased in Italy, in particularly at secondary and undergraduate level.

Mathematics&Real life (*M&R*) is a national project promoted in 2005 by the Department of Mathematics and Informatics of the University of Perugia with the purpose of stimulating a deep didactic innovation.

The experimentation, mainly addressed to secondary schools, involves students from eleven to eighteen years old.

The central point of the proposal is a dynamic interaction between the real world and the mathematic world. M&R's key word is: **education to mathematical modelling**.

The project does agree with PISA (Programme for International Student Assessment - promoted by OCSE) report themes.

Main M&R activities:
- Didactic Labs hold in different schools at a national level
- Courses addressed to teachers and tutors in order to project the didactic Labs
- Annual meeting with the active participation of students and their tutors
- Periodical supervising process of the learning activity
- Final test carried out, contemporaneously in all schools.

M&R numbers:

Hear	Labs	Schools	Teachers	Students
2005-06	21	42	45	1.321
2006-07	74	47	90	2.515
2007-08	78	48	94	2.505
2008-09	82	52	104	2.600

M&R educational materials:
- Matematica&Realtà, Laboratori di Innovazione didattica, Università degli Studi di Perugia (2006) pgg.265
- Esperienze a Confronto - Proceedings 1998-2009, DVD
- Teachers courses www.matematicaerealta.it
- Students lessons www.matematicaerealta.it
- Test 1998-2009 www.matematicaerealta.it
- Alice&Bob – Centro ELEUSI, Univ. Bocconi, M&R dossiers

Students overcoming blockages while building a mathematical model; exploring a framework

Sanne Schaap*, Pauline Vos #, M. J. Goedhart*

*University of Groningen, Netherlands

#University of Amsterdam, Netherlands

Many questions in our society are answered by using mathematical models (e.g. a formula). The complex process of building, using and refining a mathematical model is termed as modelling. An important phase in the modelling process is the translation of a phenomenon from reality into mathematics, an activity known as mathematisation, Although modelling is a compulsory topic for pre-university science-stream students in the Netherlands, students are mostly confronted with ready-made mathematical models. As a consequence, students are not well able to mathematise autonomously, i.e. to create mathematical models (Vos, 2007). Therefore, our research aims at developing components for an instructional strategy that will contribute to students' improved proficiency in mathematising, extending the current knowledge base on modelling competencies, in particular on students' blockages in the process of mathematising and ways to overcome these blockages. In this paper we search for components of a framework for identifying blockages and opportunities while mathematising (as part of modelling).

Different authors have characterised modelling as a cycle of activities (Blømhoj & Jensen, 2007; Blum & Leiß, 2005; Galbraith & Stillman, 2006). Galbraith and Stillman (2006) used their modelling cycle as a framework to identify the occurrence or removal of blockages in the modelling process. We will conceive their observation that the framework is *"yet to be refined and tested"* (p. 146) as an invitation to use, specify and extend their approach, but then limited to the subset of activities needed for mathematising (understanding, interpreting the context, structuring, simplifying, assuming, formalising, etc.).

We administered task-based interviews to three pairs of pre-university science stream students (16-17 years), who solved four tasks, in which a mathematical model was yet to be developed. This paper reports on observed blockages and opportunities and to what degree these aligned with the modelling cycle.

A number of blockages could be described using the cyclic modelling framework. However, all observed opportunities and a number of alternative blockages required an additional vocabulary for identification. Hence, we propose a framework that extends the modelling cycle with terms, such as *problem solving strategies*, *metacognition* and *beliefs* (Maaß, 2006; Meijer, Veenman, & Hout-Wolters, 2006; Polya, 1988; Schoenfeld, 1992).

Modelling in the classroom – motives and obstacles from the teacher's perspective

Barbara Schmidt

University of Education, Freiburg, Germany

Modelling is not only written into educational standards throughout Germany; other European countries also stipulate the integration of reality-based, problem-solving tasks into mathematics at school. In reality, however, things look quite different: across Europe maths lessons are still dominated by exercises in simple calculation. Why is this the case? What is preventing teachers from introducing modelling? What would motivate them? In order to explore this issue in depth, a questionnaire was developed.

The questionnaire covered with 12 scales teachers' points of view regarding different aspects of modelling. (e.g. modelling tasks are very complex; pupils reject modelling tasks; pupils work more independently with modelling tasks, modelling tasks take too much time etc.). A further aim of the questionnaire was to assess if modelling items acted more as hindrance or motivation.

In order to support and enhance the results of the questionnaire, 5 teachers were interviewed before, during and after the training course. Of particular interest were the issues that arose beyond the framework of the questionnaire and whether these were expressed more as a hindrance to or motivation for modelling. Furthermore, a prototypical process from non-modeller to modeller was to be established.

This paper intends to introduce the project LEMA, the development of the questionnaire and the survey design. Finally, the results of the questionnaire and the interviews will be presented.

Pre-service physics teachers' difficulties in constructing mathematical models of simple harmonic motion

Aysel Sen-Zeytun, Bulent Cetinkaya, Ufuk Yildirim, Ayhan Kursat Erbas

Middle East Technical University, Turkey

The content and the way the physics and mathematics taught at the tertiary level depend heavily on procedural understanding of mathematics. Thus, physics students, particularly pre-service physics teachers, lack a coherent conceptual mathematical understanding, which also hinders developing conceptual understanding of physics. Students have various difficulties in linking the mathematical processes and physics (Woolnough, 2000). To overcome these difficulties and build an understanding of critical ideas in physics can be achieved through modelling activities (Carrejo & Marshall, 2007). On the other hand, even though modelling is an effective tool, it involves a complex process, during which students may encounter problems. The purpose of this study was to understand mathematical difficulties and problems in modelling process that pre-service physics teachers confront in constructing mathematical models of simple harmonic motion.

The study took place in an elective course, *Laboratory Experiments in Science Education*, designed for pre-service physics teachers. There were six students enrolled in the course, all of whom have participated in the study. Working in pairs the participants were asked to complete a model-eliciting activity developed by the researchers according to the design principles proposed by Lesh, Pole, Hoover, Kelly and Post (2000). Data were collected from multiple sources: The audio and video recordings of each pair during the modelling activity, audio-recorded interviews with each individual student upon completion of the activity, students' reflection papers and worksheets.

The analysis of data revealed that pre-service teachers faced several difficulties during the modelling process. Identifying and simplifying the problem situation have taken a considerably long time for them. Furthermore, they did not attend to the verification step of the modelling process. We also found that students have inadequate mathematical knowledge, especially on transition between proportion and equation, interpreting graphs of functions, and graphing skills. These difficulties obstructed the construction of a mathematical model – formula for the period of a simple harmonic motion. The participants came up with a proportional relationship rather than an equation depicting the mathematical relationship among the period, mass and spring constant. The implications of these findings will be discussed.

Physics teachers' mathematical modelling in mechanics tasks

Zahra P Shirazian, Pamela Mulhall, Anthony Jones

University of Melbourne, Australia

Many secondary school students are reported as having difficulty with understanding different physics concepts involved in motion. A critical issue appears to be how students are taught to meaningfully employ the mathematics involved in solving motion tasks.

This study explores aspects of teachers' content knowledge (CK) and pedagogical content knowledge (PCK) with respect to using and understanding physics formulae, teaching those formulae and their feedback to students' written response to introductory mechanics tasks.

In an in-depth '*instrumental case study',* a comprehensive questionnaire and a problem-centred interview were used to obtain data from five teachers who taught Year 11 and 12 physics. This data was subjected to a '*qualitative content analysis*.

Findings so far show that there was a considerable range of CK and, PCK among teachers. While the teachers attempted to solve and teach the tasks in various ways and applied different mathematical modelling. It appeared that formulae were to be taken as a given, not something to be understood. When teachers gave feedback to students' responses, they were giving many alternative explanations without considering which would be most helpful in meeting students' needs.

The findings of this study have implications of the focus and design of pre-service and in-service teachers' education programs, in terms of linking the mathematics and physics concepts.

Two-stage modeling: entertaining intermediate representation

Pavel Shmakov, Natalia Slepova

University of Helsinki, Finland

"Students are much more likely to be motivated to learn and to retain information if they are happy and amused" (Oppliger, 2003).

In this paper the possibility of developing creative thinking during mathematics lessons by using the two-stage modeling is considered. In the initial stage there is a search for the "matrix" of the problem or its author's creation. A particular problem may be a problem that really exists in the world, or one close to reality (R-problem). In the first stage, an R-problem is transformed into the fantastic model of the unreal world (F-model). We define an F-model as an entertaining representation of the essential aspects of a problem existing in the real world which presents knowledge of that problem's content in usable form. Transformation of the «matrix» into the F-model is carried out by means of fairy-tale terms, humour, amusing hints, etc. The F-model is then presented to pupils. At this point there is an emotional reaction towards it. This reaction helps concentrate pupils' attention and engages them to create jointly with a teacher the mathematical (M) model. In the second stage the F-model is transformed into the mathematical model. The F-model allows engaging pupils with the process of mathematical modeling. During such modeling the inspiring math method (IM) is used. Over a few years, this method has developed into a teaching approach that assumes that: a) in the same F-model, entertainment is combined with a set of difficulty levels; b) during the course of F-model's construction there are conditions for emotions to be evoked; c) all pupils regardless of their abilities can participate actively in modeling process.

The third stage is carried out jointly with the pupils analyzing the M-model as an independent object. Pupils find different ways and solutions in accordance with their level of competence. In the fourth stage of this transfer the knowledge about the M-model is added to the F-model and an improvement of the F-model results.

REFERENCES:

Oppliger, P. A. (2003). Humour and learning. In J. Bryant, D. Roskos-Ewoldsen & J. R. Cantor (Eds), *Communication and emotion: Essays in honor of Dolf Zillmann* (pp.255-273). Mahwah, NJ: Lawrence Erlbaum Associates.

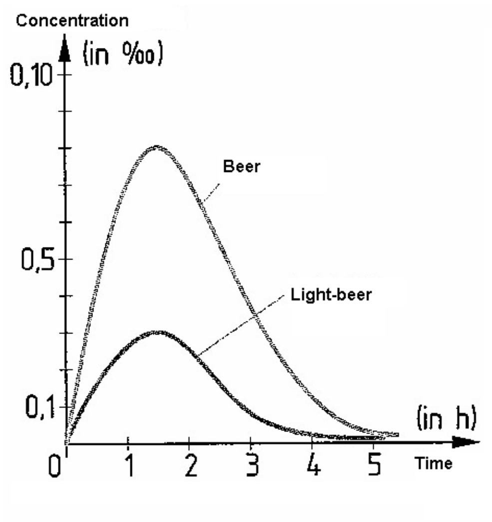
The role of real-life-mathematics in education – degradation of alcohol

<u>Hans-Stefan Siller</u>

University of Salzburg, Salzburg, Austria

In Mathematics education students can get motivated by discussing examples, which can be found in the students' life-world. Thinking about such examples is a very challenging task. If one has a look on different school-books for Mathematics education it will be possible to find different examples for different purposes. A lot of these examples want to show the possibilities of Mathematics in real-life-situations.

Looking at such examples can lead to interesting educational questions which can be discussed further if aspects of modelling are considered. The starting point for a modelling sequence is the following graphic, which can be found in the schoolbook of Malle (Mathematik verstehen 5, 2004, p.113):



By discussing these graphs it is possible to find interesting mathematical questions which can lead to a deeper understanding for procedures in our body. By modelling these graphs with students technology is an important additive. In my paper I want to show the discussion about the real-life-problem "degradation of alcohol" and its implementation in class with CASIO Classpad 300+.

Modeling and Design in Science Education

Jeroen Spandaw

Delft University of Technology, Netherlands

More or less realistic contexts have played a prominent role in Dutch school mathematics since 1985. The mathematical model is usually given and students' activities are restricted to mathematical analysis and interpretation. Far less attention is given to important modelling steps such as simplification, conceptualization, validation and successive refinement of the model. It is well known that these steps impose huge obstacles for mathematics teachers, such as domain knowledge, problems related to the philosophy of modelling and the open ended nature of modelling tasks. Since there is little long term experience in serious modelling at secondary level, we argue that one should look at work done in neighbouring fields. What can maths' educators learn from their colleagues in the natural sciences or technology? What are the lessons from modelling courses at tertiary level for students in mathematics, science and engineering? We intend to gather the experience accumulated in the fields mentioned above by interviews, video recordings of classes and analysis of teaching materials. We will analyze and summarize our findings on the basis of the state of the art literature (ICTMA 13, ICMI 14 and CERME 6).

In 2007 modelling in Dutch secondary education got a new boost by enhancing its role in the mathematics' curriculum and by the introduction of a new school subject *Nature, Life & Technology* (NLT). NLT assignments for students consist of technological design tasks and/or research projects in the natural sciences. One of the NLT modules deals with technological design, a subject which has been taught at Dutch schools since the turn of the century. These design tasks get teachers into difficulties similar to the ones mentioned above. How did teachers tackle these problems?

Another NLT module centres on dynamic modelling. Contrary to mathematics' teachers, NLT teachers have had extensive training targeted at modelling. Furthermore, one would expect that their training in science has shaped their understanding of the goals and limitations of models and has prepared them to cope with the non-mathematical steps of the modeling cycle. Does this training indeed help NLT teachers to overcome obstacles more effectively than their mathematical colleagues? If so, what are the implications for the professional development of maths' teachers?

In tertiary education, too, lecturers have gathered much experience teaching modelling. What do these experts and researchers have to say about important issues such as simplification, conceptualization, validation, authenticity, assessment and epistemological understanding of modelling? What is the role of the modeling cycle in research and teaching? What are their goals of teaching modelling, how can one achieve these goals and what are the implications for secondary education?

Pre-Service Teachers' Affinity to using Modelling and Real World Tasks in their Teaching in Years 8 -10

Gloria Stillman[1] and Jill Brown[2]

[1]*University of Melbourne, Melbourne Australia*
[2]*Australian Catholic University, Melbourne Australia*

This paper will present some of the findings from the Australian data collection of part of an international study of pre-service mathematics teachers, *Competencies of Future Mathematics Teachers*. Data related to the teaching of modelling and real world tasks in Years 8 –10 have been collected from volunteer pre-service secondary mathematics teachers from 6 cohorts at 6 university sites in three Australian states. A questionnaire and problem centred interview designed by Kaiser and Schwarz (Schwarz, Kaiser, & Buchholtz, 2008) were used for this data collection. These instruments tap into pre-service teachers' professional knowledge and thus give insight into their preparedness to teach, in this instance in the area of modelling.

The main categories of professional knowledge for teaching mathematical modelling to be reported in this paper are: (a) beliefs about the nature of mathematics, and mathematics teaching and affinity with modelling in teaching (Beliefs); (b) competencies in modelling (i.e., Mathematical Content Knowledge) which include (i) being able to mathematical model at the level of school mathematics, (ii) knowing what constitutes modelling, and (iii) knowledge of modelling processes; and(c) didactical reflections about modelling (i.e., an amalgam of Pedagogical Content Knowledge and Mathematical Content Knowledge). Other factors that may explain the variations in the responses such as mathematical background, emphases in local state curriculum documents, preparation model, whether the respondents are career changers or recent graduates from their mathematics qualifying degree, teaching experiences on practicum, and mathematics education course differences will also be reported.

Reference

Schwarz, B., Kaiser, G. & Buchholtz, N. (2008). Vertiefende qualitative Analysen zur professionellen Kompetenz angehender Mathematiklehrkräfte am Beispiel von Modellierung und Realitätsbezügen. In Blömeke, S., Kaiser, G. & Lehmann, R. (Eds.). *Professionelle Kompetenz angehender Lehrerinnen und Lehrer – Wissen, Überzeugungen und Lerngelegenheiten deutscher Mathematikstudierender und –referendare – Erste Ergebnisse zur Wirksamkeit der Lehrerausbildung.* Münster: Waxmann Verlag (p. 391 - 424).

Phases of development Mathematical Modelling activities and the Peircean Semiotic

Lourdes Maria Werle de Almeida, **Karina Alessandra Pessôa da Silva**

State University of Londrina, Brazil

In this paper we established relationships between the phases of development of Mathematical Modelling activities and the Semiotics of Charles Sanders Peirce (1839-1914), in what concerns the categorization of the signs established by that author. For this purpose, we analysed Mathematical Modelling activities and reveal how such relationships happen.

For Peirce, the Semiotics field of study was the formal doctrine of signs. The sign, by Peirce, is something which stands to somebody for something (object), in some respect of this something, but only in some form or capacity. From the study of the signs, Peirce establishes three phenomenological categories: Firstness, Secondness and Thirdness.

Firstness refers to signs that are related to chance, not seen as practical, but as a quality, a feeling, something that occurs first. Secondness refers to signs that are related to experience, to dependence's ideas, determination. Thirdness refers to signs that are related to generality, growth, intelligibility. For Peirce, the phenomenological category thirdness, corresponds to the triadic relationship in which approximates a first (firstness) and a second (secondness) and a summary is the intellectual layer of thought in signs, by which represent and interpret the world.

In the understanding that we use in this paper, during the development of a Mathematical Modelling activity, a problem to be studied is defined from a reality situation and it takes place the deduction of a mathematical model that allows to interpret the problem in mathematical language.

Therefore, the first contact is had with the real situation, establishing relationship with the phenomenological category firstness, in the sequence is determined the existence of something to be studied (a problem), which depends on the situation, establishing relationship with the secondness and, finally is generalized the situation under study using a mathematical model, it is the thirdness. The investigation that we accomplished establishes these relationships of the phases of Mathematical Modelling with Pierce's categories.

Modelling routes in 12-16 years old students

Manel Sol; Joaquín Giménez & Núria Rosich

Vilatzara School; Barcelona University. Spain

Our ongoing research, inscribed in the socio-critical perspective (Kaiser, 2007) aims to contribute to the discussion upon modelling behaviours of 12-16 year-old students showing the analysis of personal modelling routes, when students use Realistic Mathematics Social Projects (RMSP, Giménez& Sol 2005) by observing their written productions and interviews during the process.

A set of more than 30 projects were empirically analyzed by using graphs reflecting their way passing through 16 actions revised: identifying mathematical characteristics of the observed reality (A1), recognizing a social problem mathematically boarded (A2),identifying relevant objects and relations (A3), selecting right associated variables (A4), expliciting mathematical awareness about topics in which the model appears (A5), expliciting relations between real objects and mathematical content involved (A6), controlling established mathematical relations (A7), expliciting the dependence among mathematical variables (A8), expliciting a hypothesis (A9), problem posing (A10), problem solving by using different strategies (A11), finding the solutions and mathematical interpretations of them (A12), expliciting the model recognizing the meaning and limits of solutions (A13), validating the model (A14), restarting the process if the model is not satisfactory enough (A15), communicating the process and results to the colleagues (A16).

Let’s consider some patterns observed along the years: (a) Students build their own personal itineraries showing non linear modelling development (as Borromeo 2007). (b)They use personal interpretations, always as a set of problems (in which steps 11 to 13 are commonly repeated more than other steps). (c) A5,A7,A9 don’t appear. (d) The youngest students don’t pass through A6, A10 & A14, they usually go directly to A10. (e) Just in some cases we see socio-critical reasoning.

References

Borromeo, R (2007). Modelling problems from a cognitive perspective. In C. Haines, et al (eds) ICTMA12 Chichester, Horwood, 260-270.

Giménez, J. & Sol, M (2005). Student’s difficulties when starting with mathematical projects. In Mathematics Education: Paths and crossroads. Lisboa: Associaçao de Professores de Matemática (pp 217-229).

Kaiser , G (2007). Report from the WG 13. Modelling and applications.

CERME5 http://www.erzwiss.uni-hamburg.de/Personal/GKaiser/pdf-publist/cerme5-Introduction.pdf

Connecting Math and Science/Technology education, opportunities and pitfalls

Henk van der Kooij

Freudenthal Institute, Utrecht University, Netherlands

In the Netherlands, the Government wants to see math and science education at the Senior High School level (age 16-18) to be more coherent. Students (and teachers!) should experience that Biology, Physics, Chemistry and Mathematics are not isolated sciences, but often different ways of looking at the same problem situation. In the reform projects for all disciplines that started some years ago, the Context-Concept approach was accepted as the basis for the development of educational materials. Inspired by 'Chemie im Context' and 'Physik in Context' (Germany), attempts are made to connect different sciences and mathematics. In the past decade, several projects have been carried out in which Mathematics and Science (general education) or Mathematics and Engineering (vocational education) worked together on curricula with a focus on more coherence. Interesting differences between mathematics on the one hand and the natural sciences and technology on the other hand are becoming apparent on issues like ‘variables and parameters’ versus ‘quantities and constants, including dimensions and units of measurements’, ‘exact numbers’ versus ‘measures’, ‘graphs’ versus ‘diagrams’, ‘continuous’ versus ‘discrete’ and also the way models are used.

I would like to present and discuss several successes, but also mention some problems mainly regarding the attitude of mathematics teachers.

Word problem classification: A Promising Modelling task at the elementary level

Wim Van Dooren[12], Dirk De Bock[23], Kim Vleugels[2], and Lieven Verschaffel[2]

[1] K.U.Leuven, Belgium

[2] University of Leuven, Belgium

[3]Hogeschool-Universiteit Brussel, Belgium

Research (reviewed by De Bock et al., 2007) indicates that pupils do not consciously choose proportional strategies, but rather decide after a routine-based recognition of the problem type. Therefore, pupils' over-use of proportionality might be broken if pupils would pay more attention during the initial phase of the modelling process. In a task without the need to produce computational answers, pupils might engage in deeper mathematical thinking, and properly differentiate proportional and non-proportional problems. This hypothesis was tested using a word problem classification task, which is rather uncommon in the mathematics classroom.

Thirty-eight 6th graders completed a classification task and a solution task (CS-condition) and 37 6th graders got the solution task before the classification task (SC-condition). The *solution task* was a test containing 9 word problems with different underlying mathematical models (3 proportional, 3 additive, and 3 constant). For the *classification task*, pupils got 9 cards (each containing one word problem) and some envelopes. Again, 3 problems were proportional, 3 additive, and 3 constant. Problems were parallel to those in the solution task. Pupils had to search which problems belong together, put them in envelopes and write on the envelope a justification. Instructions were intentionally vague to see which criteria pupils would spontaneously use.

The results on the *solution task* showed that the proportional problems elicited, as expected, significantly more correct answers than the additive and constant problems. More importantly, pupils in the CS-condition performed significantly better on the solution task than pupils in the SC-condition. The results on the *classification task* indicated that most pupils created a group containing proportional problems, but often containing also some additive and/or constant problems, suggesting that also when classifying, pupils had great difficulties distinguishing non-proportional from proportional problems. Classifications were often imperfect, but most pupils still distinguished proportional, additive, and constant problems. While pupils' performance on the proportional problems hardly differed between conditions, their performance on the additive and constant problems was somewhat higher in the CS-condition. So, whereas the classification task enhanced later performance on the solution task, the reverse was not the case. When pupils work on a (classification) task that invites them to analyse commonalities and differences between word problems, they apparently engage in a deeper kind of mathematical thinking, which is beneficial for later problem solving, than when doing a classical word problem solving task.

Teaching mathematical modeling in Hungarian schools based on some national traditions

<u>Ödön Vancsó,</u> Gabriella Ambrus

Eötvös Loránd University, Budapest, Hungary

The authors are taking part in a Comenius EU-project (2007-09) for development a teacher training course of the topic Mathematical modeling. The project (Learning and Education in and through Modelling and Application, LEMA) is led by Katja Maas from Freiburg and six countries participated. The experience of the piloting of a planned teacher training course for mathematical modeling has shown again the well known fact that the teacher make crucial changes not easy. The participants of this course were very innovative and flexible otherwise they wouldn't have come. Against it they found very hard to change their praxis. They formulated an idea in a plenary talk. A new idea is much more "comfortable" if that is deeper imbedded in the all days praxis and seems to be more familiar and useable. If the teachers are thinking about the modeling not a totally new idea in their teaching but a thing which is known for them in their praxis, and they have to change only little details or a little bit more conscious doing it, then the teachers resist this idea less.

Among many didactical reforms in Hungary two important lines have to be chosen which are built deeply in the Hungarian tradition and both of them has important connection to modeling (of course exactly not used this word). One of them can fasten to Manó Beke who lived and worked at the beginning of XX. Century and has a friendly contact to Felix Klein the famous reformer of the end of XIX and beginning of XX Century. Another line is Tamás Varga, who is better known in abroad and overall the world, and had got interesting ideas about applications and modeling, and had a friendship to Hans Freudenthal.

In the presentation among others some of their ideas and citations of concrete old schoolbooks will be shown and a trying to build connections between the ideas of these Hungarian persons and the today used mathematical modeling. So the modeling can be seen much closer to the "Hungarian Soul" than if it seems to be a never earlier heard new idea.

Mathematical Modeling Using Open-Source Software

Kai Velten

Hochschule RheinMain, Geisenheim, Germany

Any up-to-date course in mathematical modeling must be based on appropriate software that can be used to implement the models and to solve the resulting mathematical problems. As this talk will show, freely available open-source software today covers the entire range of mathematical models that the majority of model users will ever see. A software concept is suggested that is based on a few software packages only [1]. Each of these packages is optimal in the sense of scientific excellence, continuous development and support, and wide acceptance.

Today, most modeling courses still rely on commercial software such as *Mathematica*, *Maple*, *Matlab, Comsol Multiphysics* etc., which generates a substantial amount of unnecessary costs. The prices of commercial math software for a single classroom may easily exceed a sum of 10.000 € or even 20.000 €, plus yearly costs for updates etc. Even more serious, commercial software will generate exponentially growing follow-up costs since some of the students will buy the software, and students will tend to introduce commercial math software later in their companies. Open-source software avoids these costs, and we are all responsible to remove commercial software from the classrooms whereever possible.

Beyond the cost argument, open-source based modeling courses also offer a number of practical advantages: students can use their own laptops in the courses, there is no need for the instructor to install and update 20 or more computers, courses can be held in any kind of classroom etc. Using a CAELinux-Live DVD, the entire range of mathematical open-source software is immediately available on any operating system without any installations [1].

The open source software concept suggested here is based on the following packages: *Maxima* and *wxMaxima* (maxima.sourceforge.net) for computer algebra, basic plotting, and the solution of simple ODE's; *R* (www.r-project.org) for statistics, advanced plotting, programming, advanced ODE problems, and simple PDE's; beyond this, several tools available in CAELinux (www.caelinux.com) can be used for advanced PDE problems.

[1] K. Velten. 2009. Mathematical Modeling and Simulation. Wiley-VCH.

The Effect of Mental Models ("Grundvorstellungen") for the Development of Mathematical Modelling Competencies Results of the Longitudinal Study PALMA

Rudolf vom Hofe, Alexander Jordan

University of Bielefeld, Bielefeld, Germany

TIMSS and PISA have documented considerable differences in students' achievements, including mathematics, which led to lively discussions concerning the effectiveness of the teaching of mathematics in school. However, these studies have some serious deficiencies which only partly explain the causes for differences in achievement. Studies like these are essentially producing a descriptive system monitoring concerning specific measuring moments and age groups, but due to their descriptive, cross-sectional design, they cannot provide insights into the achievement development which has led to the stated results, nor into the impact of corresponding mathematical modelling competencies and the required mental models of mathematical concepts (that we call "Grundvorstellungen").

Especially these points, however, are important for providing causes for the identified achievement deficits and evidence for possibilities for the improvement of classroom practice. Thus the aim of the research project PALMA is to pursue longitudinally students' mathematical achievement and its conditions. The essential aims are (1) the analysis of mathematical achievement development as well as corresponding modelling competencies and Grundvorstellungen from grade 5 to 10, (2) the analysis of causes of this development, and (3) providing hints for the improvement of teaching and learning of mathematics in each age group.

In our talk we will present some selected aspects of our study.

Implications of Mathematical Modelling in Pre-Service teacher Education

Michele Regiane Dias Veronez

FAFI – Faculdade Estadual de Filosofia, Ciências e Letras de União da Vitória.

A pre-service teacher has his/her first contact with mathematical modelling in college. Such contact allows the professional to understand and have some experience in modelling. This happens because in this academic environment the learners are invited to get involved in the proposed situations, and to try to solve them. In so doing, they realize the need to understand the situation under study, to verify which information is relevant, and also which mathematics or mathematical content is best represented. By means of this guidance the pre-service teacher has the opportunity to learn about the situation or theme in focus, as well as to revise, reorganize or learn mathematical concepts. At that time it is also possible to reflect about mathematical modelling in what concerns its implementation in classroom, as well as the contributions it might provide the learner. The first one allows the future teacher to get in touch with the theorizations of the teaching and learning processes, more specifically under the perspective of mathematical modelling, and to analyze the arguments which support the introduction of modelling activities in basic education (primary and secondary school).

What concerns the contributions for the learners, we could highlight the possibility of understanding mathematical concepts while analyzing diverse situations, including the ones which interest them most. In this sense this work is placed within the investigation of mathematical modelling in pre-service teacher education, and the intention is to investigate and understand how future teachers get involved with mathematical modelling, both from a theoretical viewpoint as well as regarding the modelling activities and the mathematical contents approached. To sum up, it proposes a view on the impact of experiments in mathematical modelling on teachers' education and consequently on their following practice.

What is ‘Authentic’ in the Teaching and Learning of Mathematical Modelling?

<u>Pauline Vos</u>

AMSTEL Institute, University of Amsterdam, Netherlands

In this position paper I will review different perspectives on the use of the adjective ‘authentic’ (c.q. the noun ‘authenticity’) in research papers on the teaching and learning of mathematics, in particular the teaching and learning of mathematical modelling.

In mathematical modelling, mathematics is connected to the world outside mathematics (mathematics is used, applied, adapted, interpreted) and several aspects in this process may be qualified as more or less ‘authentic’. Researchers report on authentic contexts (situations), authentic problems (tasks, assessment), authentic activities, authentic learning environments (learning situations), and so forth. The term ‘authentic’ is not only a qualification for aspects in the non-mathematical side of the modelling cycle; also the mathematical side of the modelling process can contain an ‘authentic model’, for example in the case where the model was not a simplification for educational purposes, but genuinely equal to a model from scientific research, including the ‘authentic’ computer programme used.

The term ‘authentic’ has proved useful to criticize pseudo-realistic, ‘dressed-up’ constructs of abstract mathematics, which ask students to carry out unwarranted activities. Examples will illustrate the variation of usages, in which ‘authentic’ is delineated with terms such as ‘realistic’ (from real-life, or daily life), ‘experientially real’ (imaginable, plausible, credible, conceivable), ‘accessible’ (familiar) and ‘worthwhile’ (meaningful, relevant, interesting). Some authors define degrees of ‘authenticity’ to measure the extent, to which out-of-school life is *simulated* by constructs. Others refer to a definition by Niss (1993), who linked ‘authenticity’ to the authorization by experts, aligning with research disciplines such as Archeology, where a discovered artifact may be original or a copy (i.e. a forgery) and ‘authenticity’ can only be attributed after extensive study by experts in the field.

In the Mathematics Education literature, the qualification of ‘authenticity’ is used to connect educational activities to out-of-school entities. In this paper, I will demonstrate the intrinsic difficulties in defining ‘authenticity’, because it is a social construct; trying to define it unambiguously denies differences between research communities and classroom communities. To resolve phenomenological issues, I propose an operational definition of ‘authenticity’ in Mathematics Education, and in particular in Modelling, by using the word ‘authentic’ for themes, resources, and activities, which are unmistakably *“not primarily created for educational goals”*. As such, we have to face that education always adds elements of un-authentication, only for the sake of creating better learning.

Mathematical modelling of environmental-scientific problems concerning risk analysis using Geographical-Information-Systems and self-made web applications

Ralf Wagner, Engelbert Niehaus

University of Koblenz-Landau, Landau, Germany

In mathematics, there is often a lack of interesting and authentic application contexts. These are urgently needed in order to demonstrate the importance of this subject for very many areas of modern science and research to students. Authentic problems normally have a higher complexity, which the learning abilities of the pupils exceed for the first time. At this point begins the concept of the project "Mathematisches Umweltlabor" (mathematical environmental laboratory), which was developed at the University of Koblenz-Landau, Campus Landau. In this project, complex problems are handled by the modelling in a heterogeneous team.

The processing in this project is application-oriented and handles with authentic problems from the field of environmental sciences, in particular risk analysis, which is done by students of environmental sciences, teacher students with mathematics as a major subject, and finally pupils with special talents on mathematics and sciences. Due to the heterogeneity of these participants, a mutually enriching and supplementing is possible.

An exemplary problem is the computation of a risk statement for a larger geographical area using the risk values at specific locations. The processing starts with the analysis of the problem and the consideration of the mathematical tools which are necessary for the solution. In doing so the specific interdisciplinary needs will be taken into account with the help of the environmental scientists. For the solution some software, especially computer-algebra systems and geographical information systems, is used, whereby the participating pupils are familiarizing themselves with authentic scientific tools and make use of them in a modelling context.

After the mathematical modelling and consideration of different approaches there should be an application-oriented preparation of the findings to make it possible for others to use them. This is done by creating suitable web interfaces with the possibility of data input and output and a corresponding provision of the solution. For example, a risk surface for the relevant area is automatically generated using discrete input values and is provided for further calculations. At this point, an interdisciplinary combination of mathematics, environmental sciences and computer science is obvious and the parties can recognize that the various sciences cannot sharply be disconnected to get adequate solutions to complex problems. In this contribution the interdisciplinary proceeding and the didactical concept of the "Mathematisches Umweltlabor" will be described regarding the modelling of the mentioned risk surface and the obtained results will be shown.

Understanding professional development for modelling as a collateral Transition

Geoff Wake

University of Manchester, UK

The design of mathematics curriculum, assessment and teacher development in European nations is being developed, albeit relatively slowly, partly in recognition that the application of mathematics is at the core of economic and societal activity in our technology and information rich society but also in response to international comparisons of the performance of students. The trans-European project LEMA (Learning and Education in and through Modelling and Applications) has sought to support professional development for modelling by developing materials that can be adapted for use in the different national contexts of six partnership nations.

Arising from this project, this paper seeks to explore and theorise teacher development in relation to applications of mathematics, and in particular attempts to understand why this is often problematic. A socio-cultural analysis focuses on the different activity systems or communities of practice that the teachers operate in with teachers conceptualised as “boundary-crossers” as they move between professional development activities out of school into their classrooms. This is extended further drawing on the construct of consequential transitions as developed by Beach (2007) who argues that transfer should be viewed as *knowledge propagation* “at the interface of persons and activities”. Here I suggest that teachers need to develop a new identity in relation to teaching and mathematics as they make a collateral transition.

Whilst these ideas are applicable to professional development in general, in this paper I seek to explore the issues in relation to mathematical modelling and applications in particular drawing on five key aspects of mathematics as identified by Cardella (2008) when considering the learning of mathematics for engineering: knowledge base, problem solving strategies, mathematical practices, use of resources and beliefs and affects. This leads to a theoretical framework that can be used to conceptualise professional development in relation to modelling and applications which will be illustrated using case study data from a second wave of development arising from LEMA in England.

REFERENCES

Beach K.: 2007. Consequential Transitions: A Developmental View of Knowledge Propagation Through Social Organizations, in Tuomi-Grohn, T. and Y. Engestrom (Eds), Between School and Work: New Perspectives on Transfer and Boundary Crossing (pp. 39-61). London: Pergamon.

The role of technology **in the modelling circle**

Jens Weitendorf

Gymnasium Harksheide, Norderstedt, Germany

The use of technology is often helpful or sometimes even necessary in the modelling process. Some problems are solved faster or are even trivialized; others can only be solved by using technology. In connection with this, it is important to discuss the influence of technology on the modelling circle. The question is whether technology only helps dealing with complex formulas or whether technology is even helpful in the process of understanding the problem

Therefore, the relationship between reality and the model of reality, the step to the mathematical model, can be influenced by the type of technology that is used. For a scientific approach, you have to choose between CAS, DGS and spreadsheets.

In my paper I want to show examples for all kinds of using technology. Especially by using the CASIO ClassPad 330 you have the opportunity to use all different kinds of technology which are interconnected. For example, it is possible to use the spreadsheets together with the power of a CAS.

Exposure to Mathematical Modelling of Pre-Service Foundation Phase Teachers

Helena Wessels

Stellenbosch University, South Africa

In this paper the preliminary results of a modelling project of pre-service teachers in their Foundation Phase Mathematics Education module are discussed. The purpose of the project is to focus on mathematical competencies and more specifically on modelling competencies.

Mathematics is not necessarily a requirement for prospective Foundation Phase teachers at South African universities. At the university where I teach, not all student teachers specialising in Foundation Phase have done Mathematics up to Grade 12 level, therefore quite a number of them lack mathematical knowledge and skills as well as the confidence to teach mathematics. The motivation to include modelling tasks in the Mathematics Education modules for Foundation Phase pre-service teachers is based on the fact that modelling provides the opportunity for them to expand their existing mathematical knowledge while developing complex and authentic thinking around their existing simple mathematical concepts (Niss & Blum 1991). Through modelling worthwhile mathematics is learnt and simultaneously the competency of applying mathematics and building models is developed (Niss, Blum & Galbraith 2007).

The 180 pre-service teachers working in 54 groups of 3 to 4 each completed the first of three model-eliciting tasks in class over a number of periods totalling about 3 hours. On completion of the task, the student teachers had to do group presentations of their solutions to the class, not only explaining their model, but also explaining the process the group went through in constructing the model. Afterwards questionnaires were completed by the student teachers and interviews were conducted with selected individuals and groups.

References

Niss, M & Blum, W (1991). Applied Mathematical Problem Solving. Modelling, Applications and Links to Other Subjects: State, Trends and Issues in Mathematics Instruction. *Educational Studies in Mathematics*, 22(1), 37-68.

Niss, M; Blum, W; & Galbraith, P (2007). Introduction to Modelling and Applications in Mathematics Education. In Blum, W; Galbraith, PL; Henn, HW & Niss, M (Eds.), *Modelling and Applications in Mathematics Education. 14th ICMI Study*, 3-32. New York: Springer.

The analysis of two modelling cases

Meng-da Wu, **Dan Wang**

National University of Defence Technology, Changsha, China

This paper presents the analysis of two cases in mathematical modelling course. One is the newsboy's problem; the other is the case of optimizing the number of rainfall observation stations.

In the newsboy's problem, it should build up a model to optimizing a newsboy's stocking amount of newspaper everyday. It is a typical stochastic decision making problem. Our interested point in this case is an important economic concept—marginal utility. This concept has many important applications in different aspects, whether economy or not. It can help us make better choice when trade off between two contrary objects. By deep analysis of this case, we hope that the students will have a clear and intensive understanding of the concept.

In the rainfall observation stations optimizing problem, the rainfall observation data of 12 observation stations in 10 years are presented. Reducing the number of stations under the condition that confines the information loss to a permissible bound is the mainly goal from economical consideration. A conception of multi-linear relation will play an important role in this case. The students tend to use linear relation coefficient to resolve the problem that is whether a station should be eliminated is according to its linear relation coefficient with other stations, but it is not suitable in this case because the multi-linear relation may cover up some facts. It was resolved by multi-linear regression in our paper.

The instruction and Contest of Mathematical Modeling: An effective approach to cultivate talents with creativity

Qifan Yang, Zhiyi Tan

Zhejiang University, Hangzhou, China

Believing the best way to show importance of speculative knowledge in textbooks is to encourage students to study real problems while mastering skills to apply knowledge, Zhejiang University initiated mathematical modeling course in 1983. In this course, students are exposed to opportunities & environment configured by faculties to practise theories in textbooks to improve sense of enterprise, innovation & competition. 7 courses are now offered: 1 for Dept. of Mathematics, 1 for "Mixed Class" of Engineering, 1 for Advanced Class of Engineering Education., 3 Public Optional Courses, & 1 degree course for graduate students. More than 1000 students attend lectures & practice inside & outside class every year, with at least one research paper or report submitted. Our practice is a clear proof that math modeling & its practical education is welcomed, and its instruction & competition are inspiring learning interest and spirit of creativity & enterprise.

US and China Math Contest in Modeling (MCM) were initiated in 1985 & 1992. Due to their power in attracting students to the practice of creativity, we integrate the instruction & contest to be a series of systemic practical education. Viewed as a crucial link of creative practice, math modeling instruction is comprehensively planned & organized to spread creativity fostering to every corner: freshmen are given lectures to learn what mathematical modeling is & why it's useful; sophomores/juniors are offered courses on applied math & skills to handle various real domains' problems to attend contests; juniors/seniors will be further guided on graduation project & research practice to dig into some practical problems. From 2003, a university-wide contest is further launched to serve more students. Nowadays, students' Creative Practice based on mathematical modeling is popular all over the campus, "teaching & practicing of math modeling" is identified as the vital construction project of education by the university. The spirit of "participation & propagation 1st" is kept to involve multifarious students into various research, such as DNA sequencing, fingerprint identification, cryptology design, cicadas' resonance, computer security and Day-to-day problems including campus' street lamps' distribution, registration sys' update, lunch service's optimization & Tourism Planning. Students draw substantial benefit by autonomic learning through online search & literature consult to overcome difficulty. Mr. Q. Shen's team found partial differential equation a promising tool to describe pollution, they self-studied pollution equations to identify pollutant source's location, made depollution scheme, and were awarded the prize of the Institute for Operations Research & Management Science (INFORMS). To date, we've got Outstanding (& INFORMS) twice (1999/ 2003), Meritorious Winner (1^{st} prize) for 32 times, & Honorable Mention (2^{nd} prize) for 23 times in US MCM. Our course is granted as a National Quality Course, and the teaching Team is honored as a National Level Teaching Group by the MOE.

What Did Taiwan Mathematics Teachers Think of Model-Eliciting Activities And Modeling?

Shih-Yi Yu and Ching-Kuch Chang

National Changhua University of Education, Taiwan

This article reports the sixteen secondary mathematics teachers' perceptions and obstacles of modeling after experiencing three model-eliciting activities (MEAs, Lesh & Doerr, 2003) and designing one MEA in a nine-week course linked to a master's degree program in education for in-service teachers.

The process of the research included two stages. First, these teachers as the role of students engaged in three MEAs, such as Big Foot, Parking Lot and Volleyball problems. They divided into groups with three to five teachers and cooperatively discussed to solve one MEA in every two weeks. They also wrote reflection journals to compare these MEAs and show their understanding of modeling pedagogy. Secondly, they designed one MEA every group and used Six Principles of designing MEA (Lesh & Doerr, 2003) to evaluate these MEAs by themselves and with each other group. This evaluative process also showed their perception of mathematics and understanding of MEA. Data collections included the learning sheets that showed teachers' strategies of the three MEAs and the result of the MEA they designed, observation journals, reflection journals, questionnaires, interview reports and video tapes of the classes.

These data were analyzed and interpreted into three aspects: First, these teachers regarded modeling as a problem solving process, in particular, relate to real life situation closely and also showed the possibilities and advantages for implementing MEAs in school math classes and the approval for enhancing mathematical competencies of students. Second, they mentioned obstacles of implementing MEA included the weak connection of current school curriculum, too easy mathematical contents and the setback of system of entrance exams. Third, they also revealed obstacles of designing MEA that the principles which achieve easily are Reality Principle and The Model Construction Principle, but the other four were hardly to make it. It meant that how to promote teachers' ability to designing MEAs still be the issue to address in the future.

References

Lesh, R., & Doerr, H. M. (2003). Beyond constructivism: models and modeling perspectives on mathematics problem solving, learning, and teaching. American, Lawrence Erlbaum Associates, Inc.

Assessing modelling competencies using a multidimensional IRT-approach

Luzia Zöttl, Stefan Ufer, Kristina Reiss

University of Munich, Munich, Germany

"Modelling competencies include skills and abilities to perform modelling processes appropriately and goal-oriented as well as the willingness to put these into action" (Maaß, 2006, p. 117). Thus, modelling requires sub-competencies according to the different phases of the modelling process (Maaß, 2006). Amongst others, individual modelling competencies depends strongly on the so called degree of coverage. It indicates "which aspects of the competency someone can activate and the degree of autonomy with which this activation takes place" (Jensen, 2007, p. 143). Thus, to assess students' modelling competencies on a broad basis, it is important to provide test items requiring varying degrees of coverage. Therefore, a test should contain items which cover the whole modelling process as well as items focusing only parts of this process.

Accordingly, we developed a modelling test in the field of area and circumference of rectangles, triangles and circles which contains items of both classes. We elaborated three different types of items which focus on different sub-processes of modelling activity and thus belong to the second mentioned class. Items of the first type require sub-competencies needed to build up a mathematical model. For items of the second type pure mathematical competencies are needed. However, items of the third type ask for the interpretation of a mathematical result and the validation of a presented problem solution in respect to the underlying model. In our test we integrated also items which belong to the first class, i.e. short, but complete modelling tasks.

To assess students' modelling competencies on the basis of this test we draw on item response theory instead of working with raw scores (advantages of this approach are described in the literature). To cope with the requirements of the two different classes of items and the according item types we used a multidimensional Rasch model including sub-dimensions. This model estimates not only individual person parameters, indicating the general modelling ability of a person, but also parameters which represent an individual's strengths and weaknesses in the implemented sub-dimensions.

We present first results concerning this test instrument and the described method.

Jensen, T. H. (2007). Assessing Mathematical Modelling Competencies. In C. Haines, P. Galbraith, W. Blum & S. Khan (Eds.), *Mathematical Modelling (ICTMA 12): Education, engineering and economics* (pp. 141–148). Chichester: Horwood.

Maaß, K. (2006). What are modelling competencies? *Zentralblatt für Didaktik der Mathematik, 38*(2), 113–142.

Poster

Online Teacher Training in Mathematical Modeling

R. Bruder, U. Böhm
Technische Universität Darmstadt, Germany

Mathematical modelling is part of the national standards of education in Germany. To support teachers to foster mathematical modelling competencies in mathematics education an online inservice teacher training was developed and implemented. The structure and theoretical background underlying the course, as well as insights into the content of the half-year inservice training will be provided.